‘MODERN’ PHILLIPS CURVES AND THE IMPLICATIONS FOR THE STATISTICAL PROCESS OF INFLATION

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ABSTRACT

‘Modern’ theories of the Phillips curve imply that inflation is an integrated, or near integrated’ process. This paper explains this implication and why these ‘modern’ theories are logically inconsistent with what is commonly known about the statistical process of inflation.

Keywords: Phillips curve, inflation, stationary, integrated, macroeconomics
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1. INTRODUCTION

Russell (2011, 2013), and Russell and Chowdhury (2013) assert that ‘modern’ theories of the Phillips curve that incorporate the property that the sum of the dynamic inflation terms equal one imply the theory simultaneously predicts inflation data follows an integrated process. Or if the sum is nearly one then a near integrated processes. This paper first explains this assertion before arguing why the ‘modern’ theories of the Phillips curve are inconsistent with what we commonly believe about the statistical process of inflation.

2. ‘MODERN’ PHILLIPS CURVES

The ‘modern’ Phillips curve literature can be thought of in terms of restrictions to the reduced form of the hybrid Phillips curve:

$$\pi_t = \delta_f \, E_t\left(\pi_{t+1}\right) + \delta_b \, \pi_{t-1} + \delta_z \, x_t + \varepsilon_t \qquad (1)$$

where inflation, π_t, depends on expected inflation, $E_t\left(\pi_{t+1}\right)$, conditioned on information available at time t, lagged inflation, π_{t-1}, a ‘forcing’ variable, x_t, and an error term, ε_t, due to the random errors of agents and the shocks to inflation. Inflation is defined as the first difference of the logarithm of the price level such that: $\pi_t = p_t - p_{t-1}$ and lower case variables are in natural logarithms. There are numerous measures of the ‘forcing’ variable in the Phillips curve literature including the gap between the unemployment rate and its long-run level, the gap between real and potential output, real marginal costs, labour’s income share and the markup of price on unit labour costs. Conceptually the forcing variable represents the business cycle which is a mean reverting process and therefore I(0). If the measure of the business cycle appears to be I(1) then this is likely to be due to the bias from not accounting for breaks when applying the unit root test to the series as explained in Perron (1989).

In the purely backward looking adaptive expectations Phillips curve model of Friedman (1968) and Phelps (1967) $\delta_f = 0$ and $\delta_b = 1$. In contrast, the New Keynesian (NK) Phillips Curve models of Clarida, Galí and Gertler (1999) and Svensson (2000) agents employ rational expectations and are purely forward-looking resulting in $\delta_f = 1 - d$ and $\delta_b = 0$ where d is the rate of time discount. Finally, the hybrid models of Galí and Gertler (1999) and Galí *et al.* (2001) incorporate agents that are both backward and forward looking and $\delta_f + \delta_b = 1 - d$.

The discount rate, d, is not identified explicitly in the theories without supplementary assumptions. However, assuming risk neutral agents, a symmetric loss function around the profit maximising price and an annual real interest rate of around 4 per cent then d is approximately 0.04 and 0.01 on an annual and quarterly basis respectively. Most empirical work on the NK and hybrid models proceed on the assumption that $d = 0$. Consequently, at an empirical level all three models predict that $\delta_f + \delta_b = 1$ in equation (1) and the standard interpretation of this prediction is that the long-run Phillips curve is 'vertical'. Finally, note that Russell and Chowdhury (2013) propose the statistical process consistent (SPC) Phillips curve which is also nested in equation (1) with $\delta_f = 0$ and $0 \leq \delta_b < 1$.

3. A THEORETICAL SOLUTION

Consider one standard solution for the statistical process of inflation based on the New Keynesian Phillips curve version of equation (1) which can be written:

$$\pi_t = \alpha \, E_t(\pi_{t+1}) + \beta \, x_t + \nu_t \tag{2}$$

where ν_t is a series of 'shocks' to the inflation process. If $\alpha < 1$ as in the New Keynesian model then the standard solution for equation (2) is:

$$\pi_t = E_t \sum_{j=1}^{\infty} \alpha^j \left(\beta \, x_{t+j} + \nu_{t+j} \right) \tag{3}$$

To solve equation (3) requires us to specify the statistical process of both the shocks, ν_t, and the forcing variable, x_t. Note that there is no solution to equation (3) if $\alpha = 1$ because the future is not discounted.

Ignoring the forcing variable for simplicity and, as we have assumed $\alpha < 1$:

$$\pi_t = E_t \sum_{j=1}^{\infty} \alpha^j \nu_{t+j} \tag{4}$$

If we also assume the shocks are a serially correlated autoregressive process

$$\nu_t = \rho\nu_{t-1} + \varepsilon_t \qquad (5)$$

where ε_t is a white noise process then the solution to equation (4) is:

$$\pi_t = \frac{1}{1-\rho}\,\nu_t \qquad (6)$$

implying inflation is a white noise stationary process. This theoretical approach leads to at least one solution that contradicts the assertion that inflation is an integrated, or near integrated, process in a 'modern' theory of the Phillips curve. Furthermore, this approach suggests that inflation can follow any statistical process depending on the specification of the statistical process of the shocks to inflation.

4. AN EMPIRICAL SOLUTION

Consider the following proof-by-contradiction. Estimate a hybrid Phillips curve with ordinary least squares:

$$\pi_t = \delta_f\pi_{t+1} + \delta_b\pi_{t-1} + \sum_{i=0}^{k}\delta_{x,i}\,x_{t-i} + \epsilon_t + \qquad (7)$$

where the error terms, ϵ_t, are white noise and the 'forcing variable', x_t. Further leads and lags in inflation can be added to equation (7) to improve the dynamic properties of the estimated equation but this will not change the general proof-by-contradiction argument below. Note that if the forcing variable, x_t, is a 'truly' I(1) process then the argument below based on the correspondence between the size of $\delta_f + \delta_b$ and the statistical process of inflation is broken. However, as discussed above, conceptually the forcing variable is an I(0) and not an I(1) process and not considered further here.

Assume there are two mutually exclusive and exhaustive states of the world where inflation, π_t, is either an integrated I(1) or stationary I(0) process. Consider the first state when inflation is I(1). As the forcing variable x_t is I(0) it will not enter the asymptotics of the estimation and can be ignored. In this case, standard cointegration theory implies that $\delta_f + \delta_b = 1$. If this is not the case then $\Delta\pi_t$ would be I(1) implying that π_t is I(2) which contradicts the initial assumption that π_t is I(1). Alternatively, in the second state when inflation is I(0) then $\delta_f + \delta_b < 1$. Again if this is not the case and $\delta_f + \delta_b = 1$ then this would imply that inflation is I(1) which contradicts our initial assumption.

The logic of estimating equation (7) suggests that (i) if inflation is I(1) then $\delta_f + \delta_b = 1$ and (ii) if inflation is I(0) then $\delta_f + \delta_b < 1$. The converse is equally true. If $\delta_f + \delta_b = 1$ in the theory then inflation needs to be I(1) and if $\delta_f + \delta_b < 1$ then inflation needs to be I(0) so that the theory is 'true' in the sense of being consistent with the data. By implication if $\delta_f + \delta_b$ is very close, but not equal, to 1 in the theory then inflation needs to be a near integrated process so that the theory is 'true'.

The empirical argument here can be reconciled with the theoretical argument in Section 3. If the 'modern' theory is a valid representation of the inflation data then the range of valid statistical process for the shocks in the theoretical solution in Section 3 is not infinite but limited to a singleton set containing only inflation as an integrated I(1) process.

5. WHAT IS THE STATISTICAL PROCESS OF INFLATION?

First, inflation in the developed economies over the past 60 years appears to be bounded below around zero and above at some moderate rate. The 'bounded' nature of inflation implies it cannot be an integrated process. Second, if inflation is an I(1) process then the price level is an I(2) process. Given prices are conceptually non-negative then this means the price level has a lower boundary of zero and so the price level cannot be I(2) and, in turn, this rules out inflation as an integrated I(1) process.

Third, the rhetoric underpinning all 'modern' Phillips curve theories of inflation is that the long-run rate of inflation depends on the setting of monetary policy and inflation will vary around that long-run rate. A change in monetary policy will see inflation converge on, and vary around, a new long-run rate of inflation.

From the three points above, we should confidently expect inflation to be a stationary process around a shifting mean. And that the shifts are due to changes in monetary policy which are discrete and possibly frequent. Furthermore, we can be equally confident about the three following propositions. (i) Inflation is a stationary and not an integrated process. (ii) The sum of the estimated dynamic inflation terms must therefore be less than 1. (iii) 'Modern' Phillips curve theories where the dynamic inflation terms sum to one are inconsistent with the statistical process of inflation and are a not valid descriptions of inflation. And (iv) the statistical process consistent Phillips curve theory of Russell and Chowdhury (2011) is consistent with the data being stationary around a shifting mean although other theories of inflation may also be consistent with this statistical process for inflation.

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