7 February 2011

# Non-stationary Inflation and Panel Estimates of United States Short and Long-run Phillips Curves

**Bill Russell***


## Abstract

This paper argues that because United States inflation has been non-stationary over the past five decades the vast body of empirical research that proceeds without adequately accounting for the non-stationarity in the data is invalid. Using fifty years of United States inflation data the standard results in the Phillips curve literature are shown to be due to unaccounted shifts in the mean rates of inflation over the period. Short and long-run Phillips curves for the United States are then estimated using time series panel data techniques that account for these shifts in mean.



Keywords: Phillips curve, inflation, panel data, non-stationary data

JEL Classification: C22, C23, E31


---


* Economic Studies, University of Dundee, Dundee DD1 4HN, Scotland. Email: brussell@brolga.net. Telephone: 01382 385165. Fax: 01382 384691. Data are available at www.BillRussell.info.



I would like to thank Anindya Banerjee, Steve Bond, Martin Chalkley, Yu-Fu Chen, John Dewhurst, Hassan Molana, Natalia Ponomareva, Peter Sinclair, Colin Tilley and participants at economics seminars at Queen's University Belfast and the University of Bath and the ACE 2009 conference at the University of Adelaide for their considered comments and suggestions. I also wish to thank Issam Malki for the Bai-Perron estimates of the shifts in the mean rate of inflation and Joshan Bai and Pierre Perron for making available the Bai-Perron programme.

# Non-stationary Inflation and Panel Estimates of United States Short and Long-run Phillips Curves

## 1. INTRODUCTION

While it is difficult to examine Phillips curve theories directly, the theories make very strong predictions concerning the statistical properties of inflation and it is these predictions that we will examine empirically.[1] First, we demonstrate that the empirical methodology employed in the standard literature to examine modern theories of the Phillips curve is invalid in that it is inconsistent with our understanding of the statistical process of inflation. Furthermore, given this understanding we should logically expect the standard empirical approaches to produce severely biased estimates. This is also demonstrated. Finally we proceed to estimate Phillips curves for the United States using a time series panel data approach that is congruent with our understanding of the statistical process of inflation. This empirical methodology suggests all three 'modern' theories are not supported by the data.[2] It is also found that the important prediction of the 'modern' theories that the long-run Phillips curve is 'vertical' is only approximately true. At low to moderate rates of inflation it is shown that the long-run Phillips curve has a significant, small but most likely economically important positive slope.

A straightforward taxonomy of modern Phillips curve theories can be thought of in terms of restrictions to the reduced form hybrid Phillips curve where inflation, $\Delta p_t$, depends on expected inflation, $E_t\left(\Delta p_{t+1}\right)$, conditioned on information available at time $t$, lagged inflation, $\Delta p_{t-1}$, and a 'forcing' variable, $x_t$, and written:[3]

$$\Delta p_t = \delta_f \, E_t\left(\Delta p_{t+1}\right) + \delta_b \, \Delta p_{t-1} + \delta_x \, x_t + \varepsilon_t \qquad (1)$$

---

1 Popper (1959) argues that a theory is 'scientific' if it produces predictions that can be falsified in an empirical sense. All three modern theories are therefore 'scientific' in the sense used by Popper and this paper suggests one methodology that will empirically examine the predictions of the theories directly and therefore indirectly examine the validity of the theories themselves.

2 Modern Phillips curve theories include Friedman-Phelps expectations augmented, New Keynesian and hybrid theories. See Henry and Pagan (2004) for a clear exposition of the hybrid Phillips curve.

3 The 'forcing' variable represents excess demand and is measured in the literature in a variety of ways including the gap between the unemployment rate and its long-run level, the gap between real and potential output, real marginal costs, and labour's income share.

where the error term, $\varepsilon_t$, is due to the random errors of agents and the shocks to inflation. In the purely backward looking Friedman (1968) and Phelps (1967) (F-P) expectations augmented Phillips curve model, agents hold adaptive expectations implying that $\delta_f = 0$ and $\delta_b = 1$. In the purely forward-looking rational expectations New Keynesian (NK) Phillips Curve models of Clarida, Gali and Gertler (1999) and Svensson (2000) $\delta_b = 0$ and the $\delta_f = 1$. Finally, the more general hybrid model of Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001) assumes that there are both backward and forward-looking price setting agents and that $\delta_f + \delta_b = 1$.[4] For the long-run Phillips curve to be vertical requires $\delta_f + \delta_b = 1$ in all three models of inflation.[5]

The standard empirical literature overwhelmingly provides estimates consistent with the predicted values in the modern theories of the Phillips curve. In the NK and hybrid literatures $\delta_f$ and $\delta_f + \delta_b$ are insignificantly different from 1 respectively with a larger weighting on the forward looking term in the hybrid model. This is interpreted as forward looking agents dominating backward looking agents in the price setting process. Similarly, in the F-P literature the sum of the lagged inflation terms is also insignificantly different from 1. We return to provide an explanation of the 'overwhelmingly consistent' empirical results in the conclusion.

The vertical long-run Phillips curve is a central tenant of 'modern' Phillips curve theories of inflation since Friedman (1968) and Phelps (1967) and implies that inflation may be non-stationary around multiple long-run rates of inflation.[6] Indeed, a large measure of Friedman's success in establishing the existence of the vertical long-run Phillips curve was in predicting

---

4 Equation (1) encompasses all three modern theories of the Phillips curve and is this paper's point of departure from the standard literature. Consequently, the micro-foundations of equation (1) are not set out here but can be found in detail in the references cited for the NK, hybrid and Friedman-Phelps theories of the Phillips curve. See also Woodford (2003) and Gali (2008) for clear expositions of the micro-foundations of 'modern' Phillips curves.

5 In the New Keynesian and hybrid models $\delta_f$ and $\delta_b$ sum to the discount rate and so is close, but not equal, to 1. However, so that the long-run Phillips curve is vertical, empirical researchers either impose the coefficients to sum to one or impose this restriction following testing (for example, see Rudd and Whelan 2006).

6 The term 'non-stationary' is used in this paper to mean all statistical processes other than stationary with a constant mean and includes stationary around shifting means.

the 'breakdown' of the original Phillips curve identified by Phillips (1958). The 'breakdown' was due to changes in the expected rate of inflation associated with changes in the long-run rate of inflation and therefore concomitant with inflation being non-stationary.

Consider now the estimation of Phillips curves with the last fifty years of United States inflation data. If the forcing variable in equation (1) is strongly exogenous and stationary then our prior empirical belief concerning the estimated values of $\delta_f$, $\delta_b$ and $\delta_f + \delta_b$ depends on what we believe is the 'true' statistical process of inflation.[7] For example, if we believe inflation is stationary then $-1 < \delta_f + \delta_b < 1$. If integrated of order 1 or trend stationary then we would expect that estimates of $\delta_f + \delta_b$ to be insignificantly different from 1. Alternatively, if inflation is a stationary process around shifting means then the estimate of $\delta_f + \delta_b$ will be biased towards 1. Importantly, if the shifts in mean are frequent and/or large then estimates of $\delta_f + \delta_b$ will be insignificantly different from 1.[8] These conclusions are not affected by the choice of estimator or the inclusion of more complicated dynamics such as adding further lags in inflation to equation (1).

An important question is, therefore, what is the 'true' statistical process of inflation over the past fifty years? Inflation with a constant mean will not stand up to any serious scrutiny as this would imply that there is only one long-run rate of inflation, one expected rate of inflation and one short-run Phillips curve over the past fifty years. Also implied is the Phillips curve did not 'breakdown' as there has been no change in the long-run and expected rates of inflation. Furthermore, the long-run Phillips curve on a practical level is a single point corresponding to the combination of the unique long-run rate of inflation and a unique unemployment rate. Unless we are willing to accept what is implied by a constant long-run rate of inflation we must conclude that inflation does not have a constant mean. That is, inflation is non-stationary. It is also unlikely that inflation is truly an integrated variable as inflation in the United States (and developed economies in general) appears to be bounded below at around zero and above at some moderate rate. We can also rule out that inflation is

---

[7] This is in contrast with our prior theoretical beliefs concerning the same parameters.

[8] This is a generalisation of the Perron (1989, 1990) argument that large shifts in mean in a trend stationary process may lead to the erroneous acceptance that the data contains a unit root. That is, the estimate of $\delta_f + \delta_b$ in equation (1) is insignificantly different from 1. See also Banerjee and Urga (2005).

trend stationary unless the trend is a proxy for a systematic unidirectional change in the central bank's target rate of inflation.

The final alternative is that inflation is stationary around shifting means.[9] The dynamics of inflation in 'modern' Phillips curve theories since Friedman (1968) and Phelps (1967) start with a discrete shift in monetary policy that leads to a discrete shift in the long-run rate of inflation. In the short-run, inflation displays stationary perturbations around its long-run rate. Consequently, we may expect inflation to be a stationary process around a shifting mean where the latter represents changes in the long-run rate of inflation due to changes in monetary policy.[10]

Return now to the estimated value of $\delta_f + \delta_b$ in equation (1). If we believe that the data is stationary around frequent shifts in mean then we must simultaneously believe that estimates of $\delta_f + \delta_b$ will be biased towards 1 *unless* we account for these shifts in the mean rate of inflation in the estimation process.

Graph 1 shows United States inflation measured as the quarterly change in the natural logarithm (multiplied by 400) of the seasonally adjusted consumer price index (CPI) for the period March 1952 to September 2004.[11] A number of shifts in the mean rate of inflation can be seen in the graph. These visual shifts in mean inflation can be shown more formally by applying the Bai and Perron (1998, 2003a, 2003b) technique to estimate multiple breaks in the mean rate of inflation.[12] This technique identifies seven shifts in the mean rate of inflation and therefore eight 'inflation regimes' over this fifty year period where inflation displays a constant mean. The mean rates of inflation for each inflation regime are shown on Graph 1 as

---

9 Another alternative is that inflation is integrated of order greater than 1. However, as the inflation data appears bounded this alternative is excluded from this discussion for the sake of simplicity.

10 Russell (2006) makes this argument in detail. Note that the inclusion of more than one stationary forcing variable to equation (1) may improve its description of the dynamics of inflation. However, as a linear combination of stationary forcing variables cannot explain a non-stationary variable (i.e. inflation) the argument remains valid.

11 See the Data Appendix for details and sources of the data used in this paper. The data is the same as that used in Russell and Banerjee (2008) to facilitate comparison.

12 Section 2 provides details concerning the estimation of inflation regimes using the Bai-Perron technique.

horizontal solid thin lines. From a purely visual perspective, the Bai-Perron technique appears to have identified all the large shifts in mean inflation over the period.

In the next section we demonstrate that estimating Phillips curve models without taking into account that inflation is non-stationary leads to biased estimates of $\delta_f$ and $\delta_b$ and, in turn, incorrect inferences concerning the underlying behaviour of economic agents. It is also demonstrated that the estimates of $\delta_f$ and $\delta_b$ in the Phillips curve literature are a direct result of estimating the models with non-stationary inflation data. Consequently, the empirical results published in the standard literature over the past thirty five years that do not account for the non-stationary properties of the inflation data are 'spurious' in the sense of Granger and Newbold (1974) and Perron (1989).

Having demonstrated that the standard empirical results are due to unaccounted shifts in the mean rate of inflation, Section 3 estimates Phillips curve models of United States inflation using panel estimation techniques that allow for these shifts in mean. Once the shifts in mean are accounted for, the panel estimates suggest that (i) there is no significant evidence that expected inflation as commonly measured in the standard NK and hybrid Phillips curve literatures influences inflation; (ii) lagged inflation has a coefficient significantly less than 1 which is inconsistent with the 'strict' adaptive expectations version of the Friedman – Phelps Phillips curve model of inflation; (iii) the long-run Phillips curve has a small and significant positive slope in line with the views expressed in Akerlof, Dickens and Perry (1996, 2000), Chen and Russell (2002), Friedman's (1977) Nobel lecture, Ross and Wachter (1973), Russell (1998), Russell and Banerjee (2008) and Russell, Evans and Preston (2002); and (iv) there is evidence that the long-run Phillips curve is non-linear and becomes steeper with higher levels of mean inflation.

## 2. THE STANDARD ESTIMATES OF PHILLIPS CURVES ARE BIASED

To demonstrate that the estimates of the coefficients are biased and due to overlooked non-stationarity in the data we estimate the hybrid and F-P Phillips curve models using Monte Carlo techniques.[13] We begin by generating a random variable, $z_t$:[14]

$$z_t = ran\left(0, \sigma\right)_t + \mu_t^i \tag{2}$$

where $ran\left(0, \sigma\right)$ is a random draw from a normal distribution with mean zero and standard deviation, $\sigma = 1.665956$, and $\mu_t^i$ is the value of the mean rate of United States CPI inflation in regime $i$ identified by the Bai and Perron (1998, 2003a, 2003b) technique and shown in Graph 1. The standard deviation is equivalent to that of the United States inflation data de-meaned for the identified shifts in mean inflation.[15]

The Bai-Perron technique estimates the dates of $k$ breaks in the mean rate of United States inflation, $\Delta p_t$, by minimising the sum of the squared residuals in the structural breaks model:

$$\Delta p_t = \gamma_{k+1} + \tau_t \tag{3}$$

where $\gamma_{k+1}$ is a series of constants that represent the $k+1$ 'inflation regimes' and $\tau_t$ is a random error. The minimum distance between breaks is assumed to be ten quarters and the final model is chosen using the Bayesian Information Criterion. The estimated United States inflation regimes are reported in Table 1 and shown on Graph 1.

---

13 The New Keynesian model restricts $\delta_b = 0$ in the hybrid model. As this restriction is comprehensively rejected by the United States data the New Keynesian estimates are not reported so as to simplify the exposition.

14 The random data is generated using WinRATS pro 6.2 with a 'seed value' of 171293.

15 Generating $z_t$ 10,000 times the mean value of the ADF test statistic assuming a constant and four lags is -2.56 with a critical value of – 2.88 we would accept the null of a unit root in $z_t$. The ADF test statistic of $z_t$ without the shifts in mean is stationary with an ADF test statistic of – 6.45.

The ‘mean-shift’ inflation series, $z_t$, is used to replace the dynamic inflation terms in the Phillips curve models where the dependent variable remains actual inflation, such that:

$$\Delta p_t = \hat{\delta}_f \, E_t(z_{t+1}) + \hat{\delta}_b \, z_{t-1} + \hat{\delta}_U \left(U - U^*\right)_t + \omega_t \qquad (4)$$

The forcing variable in these models is the gap between the actual unemployment rate, $U_t$, and the potential unemployment rate, $U_t^*$, and is measured as the United States unemployment rate adjusted for a broken trend in June 1978. To conform to the recent hybrid Phillips curve literature, the models are estimated using GMM with instruments of three lags of both the generated mean-shift variable, $z_t$, and the de-trended unemployment rate, $\left(U - U^*\right)_t$.[16] The hybrid Phillips curve encompasses both the F-P and NK models with a single lead and a single lag in inflation. The Friedman-Phelps model is estimated with three lags of inflation. In both models the number of lags of inflation is chosen by a 5 per cent *t* criterion.

Note that in equation (4), the only information contained in the explanatory variables that is relevant to explaining the dependent variable (which is actual inflation) is the shift in mean contained in $z_t$ and the forcing variable, $\left(U - U^*\right)_t$. The hybrid and F-P models are estimated 10,000 times using Monte Carlo techniques so as to recover the mean values of the estimates.

The mean values of the estimates from the hybrid and F-P versions of equation (4) are reported in Table 2 and are similar to the estimates reported in the standard Phillips curve literature.[17] In the hybrid model, the estimated coefficient on expected inflation is 1.2751 and the coefficient on lagged inflation is insignificant and – 0.0927. The sum of the coefficients on the hybrid and F-P models are insignificantly different from 1 (hybrid: 1.1824 and F-P: 0.8993).

---

[16] The results and conclusions presented here do not depend on GMM and are robust to any estimator including ordinary least squares and two stage least squares as long as the technique is only appropriate for stationary data with a constant mean.

[17] The results can be compared with those reported in Table 1 of Gali *et. al.* (2005) or any of the Phillips curve papers cited in Section 1.

It appears, therefore, that not accounting for the shifts in mean inflation leads to severely biased estimates of the dynamic inflation terms and that the shifts in mean will generate results very similar to those in the standard literature. We may conclude, therefore, that the results reported in the modern Phillips curve literature are dominated by, and are as a direct result of, the shifts in the mean rate of inflation that are not accounted for in the estimation of the models.

This experiment points strongly to why expected inflation is a significant explanatory variable in the standard hybrid Phillips curve literature. The mean-shift inflation series contains no relevant information for explaining actual inflation other than the size and timing of the shifts in mean inflation. The hybrid model in column 1 of Table 2 shows that the coefficient on expected inflation is insignificantly different from 1. This can only be due to the shifts in mean contained in the generated mean-shift inflation series. Simultaneously, the lag in inflation is insignificant. This experiment suggests the shift in the mean rate of actual inflation is explained by the expected inflation term. Having explained the actual shift in mean inflation, the mean-shift inflation data contains no further information and so the lag in mean-shift inflation is insignificant. One might hypothesise that if the lag in inflation did contain some relevant information concerning actual inflation then the lag in inflation would also be significant in the hybrid model. If this hypothesis is correct then it implies that in the standard empirical hybrid Phillips curve literature, expected inflation is significant (and large) due to the unaccounted shifts in mean inflation while lagged inflation is significant (and small) due to the information content of the inflation data other than the shifts in mean inflation.

## 3. PANEL ESTIMATES OF UNITED STATES PHILLIPS CURVES

We now proceed to estimate short and long-run Phillips curves assuming explicitly that inflation is stationary around shifting means. Based on this assumption, we partition the data into the eight inflation regimes identified using the Bai-Perron technique. We then organise the data as time series of eight individual inflation regimes. As the data are stationary in each regime by construction this allows us to analyse the data using standard unbalanced panel estimation techniques to simultaneously estimate the short-run Phillips curves for each of the inflation regimes.[18] Note that the model estimates one short-run Phillips curve for each inflation regime that has a constant mean, or long-run, rate of inflation.

Two broad panel estimators present themselves.[19] The random effects estimator is strongly rejected by the data. The fixed effects estimator accounts for the different mean rates of inflation across regimes by introducing a constant, or fixed effect, for each regime. This estimator is sometimes referred to as the 'within estimator' for it uses the within regime, and not the between regime, variance in the data. As the fixed effects in the model have a straightforward economic interpretation and the model is accepted by the data we present the fixed effects estimates below.

The panel fixed effects specification of the hybrid Phillips curve model of equation (1) can be written;

$$\Delta p_t^i = \phi^i + \phi_f \, E_t^i \left(\Delta p_{t+1}^i\right) + \phi_b \, \Delta p_{t-1}^i + \phi_U \, U_t^i + \eta_t^i \qquad (5)$$

where the '*i*' superscript indicates the inflation regime that the data is drawn from and the forcing variable is the rate of unemployment, $U_t^i$. The unobserved regime-specific time invariant fixed effects, $\phi^i$, allow for shifts in the mean rate of inflation across regimes and $\eta_t^i$ is a disturbance term which is independent across inflation regimes.[20] The hypothesis that the coefficients $\phi_f$, $\phi_b$ and $\phi_U$ are the same across regimes cannot be rejected by the data leading to the restricted model in equation (5) (see the notes to Table 4).

Panel estimation of Phillips curve models allows us to; (i) estimate the fixed effects, $\phi^i$, that allow for different mean rates of inflation in each inflation regime; (ii) estimate the coefficients on expected, $\phi_f$, and lagged inflation, $\phi_b$, to examine the veracity and size of

---

18 I thank Hassan Molana who suggested during a conversation concerning the identification of inflation regimes that, once inflation has been transformed into a stationary process with a constant mean in each inflation regime, panel data techniques are a valid estimation procedure.

19 See Smith and Fuertes (2003) for an excellent introduction to time series panel estimation and Baltagi (2001) and Hsiao (2003) for more detailed studies.

20 Conceptually this assumption is likely to hold as the time periods are not aligned across inflation regimes.

the forward and backward looking behaviour of agents; and (iii) estimate the short-run Phillips curve for each of the eight inflation regimes.

It is likely that inflation and the rate of unemployment are determined simultaneously and so the contemporaneous unemployment rate is endogenous in the estimated model. Furthermore, our measure of expected inflation suggests it will be correlated with the error term. We address these problems by estimating the fixed effects model using two stage least squares (2SLS) where the instruments are two lags of inflation and the unemployment rate.

The panel approach outlined here has two major advantages over a standard time series approach that estimates the models after de-meaning the data with a series of shift dummies. First, the instruments used in the estimation conceptually represent the formation of expectations by agents. Reorganising the data into cross section time series panels (regimes) allows the expectation formation at the start of each regime to be based only on information from within the same regime and not information from the end of the previous regime as would be the case with the time series approach with dummies. Second, the panel approach allows the removal of regimes from the estimation process that we suspect are non-stationary following the Bai-Perron identification of the breaks. This is not possible if we estimate the models using the standard time series approach with dummies.

### *3.1 Panel estimates of United States short-run Phillips curves*

Reorganising the data as an unbalanced time series panel does not in itself affect the standard results of the Phillips curve literature. This is easily demonstrated by estimating equation (5) with the constant, $\phi^i$, restricted to be the same across all eight inflation regimes. This is equivalent to estimating the model assuming a constant mean rate of inflation across all the inflation regimes as in the standard empirical literature. Two stage least squares estimates of the hybrid and Friedman-Phelps Phillips curve models with the restricted constant are provided in Table 3. Note the results are similar to those reported for the respective models in the standard Phillips curve literature. For the hybrid model, the sum of the estimated coefficients on the lead and lag of inflation is insignificantly different to 1 ($\phi_f + \phi_b = 0.9693$) and that there is a significant role for both forward and backward looking agents. For the F-P model the sum of the estimated coefficients on the three lags in inflation is also insignificantly different from 1 ($\sum \phi_b = 0.9810$).

Two stage least squares panel estimates of the fixed effects hybrid model are reported in column 1 of Table 4. Further lags in inflation and the unemployment rate are insignificant (see notes to Table 4). Once the shifts in mean are accounted for we are unable to identify a significant role for expected inflation in the inflationary process and the sum of the estimated coefficients on the dynamic inflation terms is 0.4736 which is significantly less than 1.

Excluding the insignificant expected inflation term, the estimated Friedman-Phelps model is reported in column 2 of Table 4. Further lags in inflation and the unemployment rate remain insignificant (see notes to Table 4). The estimated coefficient on lagged inflation is now 0.3323 which remains significantly less than 1. The unemployment rate is significant and negative with a value of - 0.3159 which suggests that the short-run Phillips curve for each inflation regime has a significant negative slope as we might expect.

The Bai-Perron technique results in two of the estimated inflation regimes (numbers 4 and 5 reported in Table 1) being defined by the minimum quarters between breaks constraint. Consequently the means of the data in these two regimes may not be constant in a statistical sense. To examine whether these two regimes are in some way ‘driving’ the results reported in Table 4, the models were re-estimated with these two regimes excluded. The results are not affected in any meaningful way. In the hybrid model, expected inflation remains insignificant and both the sum of the dynamic inflation terms, $\phi_f + \phi_b = 0.4929$, and lagged inflation, $\phi_b = 0.3133$, are significantly less than 1. In the F-P model the coefficient on lagged inflation is significantly less than 1, $\phi_b = 0.3133$, and the unemployment rate is significant and negative, $\phi_U = -0.3135$. The results are very similar to those reported in Table 4 in terms of estimated coefficients and the diagnostics of the model.

### *3.2 Alternative Measures of the Forcing Variable*

Some observers may wonder if the models are robust to alternative forcing variables reported in the literature. To this end the models were re-estimated with the output gap and the unemployment rate gap as alternative forcing variables. Each alternative was measured in two ways. The first uses an unrestricted linear trend and constant to de-trend the forcing variable in each regime of the panel format of the data.[21] The second uses a Hoddrick-Prescott filter to de-trend the time series data prior to partitioning the data into regimes for the panel estimation.

The 2SLS fixed effects hybrid and Friedman-Phelps models with the alternative forcing variables are reported in columns 1 to 4 and 5 to 8 respectively in Table 5. The results are very similar to those set out in Table 4 with (a) the lead in inflation insignificant in all hybrid models (b) the lag in inflation is significant and very similar in numerical size to the original estimates in both the hybrid and F-P models; and (c) the additional forcing variables have the 'correct' sign and are significant at the 5 per cent level for the F-P models and unemployment gap versions of the hybrid model, and at the 10 per cent level in the output gap versions of the hybrid models.

### *3.3 Estimating the Long-run Phillips curve*

The models based on equation (5) provide estimates of one short-run Phillips curve for each of the eight inflation regimes. In turn, each short-run Phillips curve is associated with one long-run rate of inflation and one long-run rate of unemployment. Assuming the long-run rate of inflation is equal to the mean rate of inflation, $\overline{\Delta p}^{i}$, in each regime the long-run unemployment rate for inflation regime *i*, $\tilde{U}^{i}$, can be calculated from the estimates of equation (5) as:

$$\tilde{U}^{i} = \frac{1}{\phi_U}\left[\overline{\Delta p}^{i}\left(1-\phi_f-\phi_b\right)-\phi^{i}\right] \tag{6}$$

Furthermore, in the fixed effects model $\phi^{i} = \overline{\Delta p}^{i}\left(1-\phi_f-\phi_b\right)-\phi_U\overline{U}^{i}$ which means the long-run rate of unemployment is equivalent to the mean rate of unemployment, $\overline{U}^{i}$, such that $\tilde{U}^{i}=\overline{U}^{i}$.[22] If we also assume the long-run rates of inflation and unemployment from each

---

[21] To de-trend the forcing variables with a linear trend we estimate: $x_t^i = \delta^i + \delta_T^i\, Trend^i + \psi_t^i$ where $x_t^i$ is the forcing variable and the estimated residual, $\hat{\psi}_t^i$, is the de-trended forcing variable in each regime.

[22] The mean rates of inflation and unemployment are calculated for each regime excluding the first two observations to account for the instruments.

inflation regime lie on the long-run Phillips curve then the locus of eight combinations of long-run rates of inflation and unemployment will loosely identify the long-run Phillips curve.

Table 6 provides three estimates of the long-run Phillips curve from the eight combinations of the mean rates of inflation and unemployment. The first is the linear long-run Phillips curve, $\overline{\Delta p}^{i} = \delta_0 + \delta_1 \overline{U}^{i}$, which shows the long-run curve has a significant positive slope. However, if the long-run Phillips curve is linear and not vertical then as mean inflation increases the rate of unemployment would eventually become greater than 1 with a positive slope. This is outside the possible range for the unemployment rate and therefore the 'true' long-run Phillips curve cannot be linear over the full range of inflation if the long-run curve is not vertical.[23]

We therefore provide estimates of a power model, $\overline{\Delta p}^{i} = \alpha_1 \overline{U}^{i\alpha_2}$, and an exponential model, $\overline{\Delta p}^{i} = \beta_1 \exp^{\beta_2 \overline{U}^{i}}$, of the long-run Phillips curve. Both non-linear models are estimated in their linear forms and reported in Table 6. Based on the $R^2$ criteria, the standard error of the estimate, and the argument that the 'true' long-run Phillips curve cannot be linear if it has a positive slope, the power non-linear model is the better of these non-linear descriptions of the long-run Phillips curve. All three estimates of the long-run Phillips curve reported in Table 6 suggest the curve has a small significant positive slope where higher mean rates of inflation are associated with higher long-run rates of unemployment.[24]

The linear long-run estimates reported in Table 6 can be compared with those of Russell and Banerjee (2008). They also argue that the 'true' statistical process of inflation is stationary around frequent shifts in mean but that this can be approximated by an integrated process. Using the same data as in this paper they estimate the long-run United States Phillips curve (in the sense of Engle and Granger 1987) as $\Delta p_t = \delta_1 + 2.714 U_t$ which displays a similar positive slope to the linear long-run Phillips curve derived from the panel estimates and reported in Table 6 of $\overline{\Delta p_t} = \delta_2 + 1.3998 \overline{U_t}$.

---

[23] However, over the range of inflation experienced by the United States (and other developed economies) over the past fifty years the long-run Phillips curve may be approximately linear.

[24] Russell and Banerjee (2008) provide a brief overview of the literature on positive sloping long-run Phillips curves.

It appears, therefore, that what is important when identifying the long-run Phillips curve is to first recognise that the data are non-stationary and then estimate the long-run Phillip curve in such a way that it does not bias the estimates nor impose invalid restriction on the data. An example of the former is estimating the models without accounting for the shifts in the mean rate of inflation. An example of the latter is differencing the inflation data in response to an erroneous belief that the data is integrated.[25]

### *3.4 A visual representation of the short and long-run Phillips Curves*

Graph 2 provides a visual representation of the panel estimates of the short-run Friedman-Phelps Phillips curves reported in column 2 of Table 4. The thin negatively sloped lines marked SRPC 1 to SRPC 8 are the estimated short-run Phillips curves for each of the eight inflation regimes once the short-run inflation dynamics are accounted for. Each short-run curve is drawn for the observed, or effective, range of unemployment rates in the respective inflation regime. Also shown on the graph are the actual combinations of inflation and the unemployment rate for the eight inflation regimes with the data from each regime represented by a different symbol. Shown as large crosses on the graph are the combinations of the long-run rates of unemployment and inflation for each inflation regime. The solid line with a positive slope labelled LRPC is the estimated power model of the long-run Phillips curve reported in Table 6. Note that the effective range of the short-run Phillips curves are to the left of the long-run curve when the mean rate of inflation is rising (i.e. regimes 3, 4 and 6) and to the right when the mean rate is falling (i.e. regimes 5, 7 and 8). This is exactly as predicted by the modern theories of the Phillips curve.

## 4. IMPLICATIONS OF THE EMPIRICAL RESULTS

In the standard Phillips curve literature, finding the sum of the estimated coefficients on the dynamic inflation terms equal 1 is taken as important evidence that the underlying behavioural theories are correct and that the long-run Phillips curve is vertical. The behavioural emphasis on the estimated coefficients on the dynamic inflation terms is entirely

[25] Differencing the data imposes a coefficient of one on lagged inflation. This restriction may be accepted by the data but if the estimates are biased due to unaccounted shifts in mean inflation then the restriction rigidly imposes the bias on the data and the model.

misplaced. For example, the hybrid Phillips curve papers of Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001) use the estimates of $\delta_f$ and $\delta_b$ in equation (1) to choose between the competing Phillips curve models on the basis of the veracity of the different underlying behavioural assumptions of each model. If the size of the estimated coefficients is due to the important 'model-defining' behaviour of economic agents then this behaviour should not only be present over the whole sample but also during all sub-samples. In other words, if the behaviour is present and stable then the estimated individual coefficients should be stable and sum to 1. If inflation is stationary around shifting means over the past fifty years as argued here then the biased estimates of the sum of $\delta_f$ and $\delta_b$ in the standard literature are likely to be insignificantly different from 1. However, within each stationary episode (or inflation regime) when the mean is constant, we know that the absolute value of the sum of the estimated coefficients, $\delta_f + \delta_b$, must be less than 1 by the definition of stationarity.[26] Therefore, the estimates of $\delta_f$ and $\delta_b$ must be unstable when inflation is stationary around shifting means and therefore do not have a behavioural interpretation unless we accept the underlying behaviour of the models is similarly unstable.

The standard literature also interprets a finding of $\delta_f + \delta_b = 1$ in equation (1) as evidence that the long-run Phillips curve is vertical. This interpretation is similarly misplaced. Consider three possible statistical processes for inflation and the forcing variable. First, if inflation is an integrated variable and the forcing variable is stationary then the standard interpretation would be correct.[27] However, while it may at times be useful to model inflation as an integrated variable it cannot by truly integrated due to the apparent upper and lower bounds to inflation. Second, if both the forcing variable and inflation are stationary then in the long-run both variables attain their respective unique mean values. Furthermore, $\delta_f + \delta_b$ must be less than one and the data can only identify one long-run combination of inflation and the unemployment rate. Equation (1) would then be estimating one short-run Phillips curve associated with one long-run rate of inflation and one expected rate of inflation. The data in

---

[26] Akerlof *et al.* (2000), Brainard and Perry (2000) and Russell and Banerjee (2008) use three different approaches to demonstrate the instability of the estimated dynamic inflation terms in the Phillips curve model using United States data.

[27] There can be no long-run relationship in the sense of Engle and Granger (1987) between an integrated inflation variable and the forcing variable if the later is stationary.

this case does not contain any information concerning different long-run rates of inflation and therefore cannot identify the long-run Phillips curve even if it is vertical in a world with changing mean rates of inflation.[28]

The third alternative is when inflation is stationary around large and frequent shifts in mean. In this case $\delta_f + \delta_b = 1$ due to the bias irrespective of whether or not there is a long-run relationship between inflation and the forcing variable. To examine the slope of the long-run Phillips curve one needs to calculate the long-run value of the forcing variable for each and every long-run, or mean, rate of inflation. The combinations of the long-run values of the unemployment rate and inflation can then be used to identify the slope of the long-run Phillips curve. Estimates of the long-run Phillips curve derived in this way are reported in Table 6 and shown in Graph 2 and suggest that the long-run Phillips curve has a significant and important positive slope. Furthermore, there is evidence that the long-run curve is not linear and becomes steeper with higher mean rates of inflation.

Therefore, the finding that $\delta_f + \delta_b = 1$ in the standard Phillips curve literature should not be interpreted as evidence that the long-run Phillips curve is vertical nor validation of the proposed underlying behavioural theories. Instead the finding should alert the researcher that (i) the inflation data is non-stationary; (ii) the estimation technique is inappropriate; and (iii) the estimates are biased and imprecise.

## 5. CONCLUSION

The empirical Phillips curve literature reveals a strange dichotomy in the economics profession. Since the work of Box and Jenkins (1976), Granger and Newbold (1974, 1977), Hendry (1980), Phillips (1986), Plosser and Schewert (1978) and Yule (1926), on ‘spurious’ regressions, applied time series economists are careful when estimating models to deal appropriately with non-stationary data. The dichotomy is that even though applied time series economists are in general careful in this respect, nearly all of the empirical work on the ‘modern’ Phillips curve fails to adequately account for the shifts in the mean rates of inflation.

---

[28] The irony is the standard estimation techniques are now valid but the long-run Phillips curve cannot be identified.

For example, most empirical work on the 'modern' Phillips curve over the past four decades makes use of a wide range of estimators that are unbiased only if the data are stationary.[29] Even when inflation is recognised to follow a non-stationary process the estimation usually proceeds under the assumption that inflation is integrated.[30] Both statistical processes are difficult to sustain as argued above. The possibility that inflation is stationary around shifting means is not considered even though we might expect this from the modern theories of the Phillips curve.[31] Consequently, the standard literature does not adequately account for the shifts in mean inflation in the estimation process and the estimates of $\delta_f + \delta_b$ as demonstrated above are severely biased upwards. It is surprising that Phillips curve models are popularly estimated in this way without accounting for the shifts in mean inflation given that the modern Phillips curve literature along with the 'breakdown' of the original Phillips curve leads us to expect that inflation is stationary around shifting means.

This paper argues that it is legitimate to model inflation as a stationary process with shifting means. Once the shifts in mean inflation are accounted for in the estimation process, the standard results of the modern empirical Phillips curve literature of the last thirty five years disappear. In particular, (i) there is no significant role for expected inflation as commonly measured in the hybrid model (see column 1 of Table 4); (ii) the sum of the coefficients on lagged inflation is significantly less than 1 by a wide margin in the Friedman-Phelps expectations model (see column 2 of Table 4); and (iii) we can identify a small and significant positive slope to the long-run Phillips curve (see Table 6). It appears from the empirical analysis above that the standard results of the modern Phillips curve literature are due to a combination of ignoring the non-stationary properties of the inflation data and the use of inappropriate estimation techniques.

---

29 A small sample of this work includes Alogoskoufis and Smith (1991), Batini, Jackson and Nickell (2000, 2005), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001, 2005), Gordon (1970, 1975, 1977, 1997), Kiley (2007), McCallum (1976), Roberts (1995), Rudd and Whelan (2005, 2007) and Sumner and Ward (1983).

30 For example see Cogley and Sbordone (2005, 2006), Ireland (2007), King and Watson (1994) and Stock and Watson (2007).

31 Ball (2000) and Levin and Piger (2002) acknowledge that changes in the mean rate of inflation may affect the estimates of Phillips curves substantially. However, in contrast with the analysis above, these papers do not systematically account for all the shifts in the mean rates of inflation when estimating the models.

The analysis above also points to why the standard empirical Phillips curve literature produces estimates that are 'overwhelmingly consistent' with the predictions of the modern Phillips curve theories when estimated over a range of data samples and across different countries. The standard literature almost universally ignores, or poorly models, the shifts in mean inflation leading to biased estimates of the Phillips curves. Given the shifts in mean can be found in all long samples of post second world war inflation data and across all developed and developing countries the bias in the estimates reported in the standard empirical literature is universal and large. It is the universal nature of the bias and its size that leads to almost identical results in the standard literature. The consistency is not due to the identification of any consistent underlying behaviour of agents over time and across countries. Instead, the consistent empirical findings in the standard literature simply reflect the consistent application of invalid estimation techniques that do not appropriately account for the shifts in mean inflation when estimating Phillips curves.

The analysis above raises a number of questions. First, is the estimated slope of the long-run Phillips curve important? The estimated power model of the non-linear long-run Phillips curve (as shown in Graph 2 and Table 6) suggests that the increase in mean inflation during the 1970s from around 4 ½ to 11 per cent per annum was associated with an increase of around 1 ¾ of a percentage point in the long-run rate of unemployment. Shifts in unemployment of this magnitude for moderate increases in inflation would appear to be important in both economic and social senses.

Second, are these empirical results in some way dependent on how the Bai-Perron technique identified the eight inflation regimes? Consider the case where the 'true' number and dates of the inflation regimes differ from those identified using the Bai-Perron technique. The identified inflation regimes used here will then contain some residual non-stationarity and there will be an upward bias in the estimates of $\delta_f$ and $\delta_b$. As the Bai-Perron technique is unlikely to have identified the 'true' number and dates of the inflation regimes we might conclude that the estimates provided above of $\delta_f$, $\delta_b$ and $\delta_f + \delta_b$ are the upper bounds of estimates based on the 'true' inflation regimes. Therefore, the estimates above that overturn the standard empirical results in the literature are not the result of incorrect identification of the inflation regimes by the Bai-Perron technique. Instead, given the Bai-Perron technique is unlikely to have identified the inflation regimes exactly, this technique makes it more rather than less difficult to overturn the standard results of the empirical Phillips curve literature which has been shown here to rely on some non-stationarity in the data.

Third, are these results inconsistent with modern theories of the Phillips curve? The answer is yes and no. The yes answer is in the following sense. Consider a period when inflation is stationary such as since the early 1990s in the United States. The strict interpretation of the modern theories requires that $\delta_f + \delta_b$ always equals 1 in equation (1). This conflicts with the initial assumption that we are considering a period when inflation is stationary which means, by definition, that the absolute value of $\delta_f + \delta_b$ is less than one. When inflation has a constant mean it appears that agents cannot conform to the Friedman-Phelps, New Keynesian or hybrid theories of the Phillips curve.

However, when there is an increase in mean inflation it does appear from the estimates reported above that economic agents adjust their expectations so that the short-run Phillips curve shifts upwards. Further empirical analysis is required to answer the difficult question of how agents adjust expectations in response to shifts in the mean rate of inflation. This upward shift may be associated with forward and/or backward looking behaviour of agents. But it is a heroic and very narrow assumption that all agents behave in only one way as in the Friedman-Phelps and new Keynesian models or in only one of two ways as in the hybrid model. It may well be that agents indulge in a variety of ways to adjust prices and their behaviour depends in part on the economic environment. In an important sense, the results reported above open the way for a richer modelling of how agents actually adjust prices.

Finally, is the remarkable Friedman and Phelps insight of the vertical long-run Phillips curve correct? The answer is ‘yes’ to a first approximation. If the long-run Phillips curve is non-linear as argued above and shown in Graph 2 then over the range of inflation from zero to infinity the long-run curve is vertical to a first approximation. However, over the smaller range of inflation experienced by the United States over the past fifty years there appears to be a significant positive slope to the long-run curve. Consequently, the Friedman-Phelps insight is only approximately true and this paper empirically demonstrates that the second order of the approximation appears to be both significant and economically important at the low to medium rates of inflation experienced by the United States over the past fifty years.

## 6. REFERENCES


Akerlof, G.A., Dickens, W.T., Perry, G.L., 1996. The Macroeconomics of Low Inflation, *Brookings Papers on Economic Activity*, 1996, no. 1, 1-76.

Akerlof, G.A., Dickens, W.T., Perry, G.L., 2000. Near-Rational Wage and Price Setting and the Long-run Phillips Curve, *Brookings Papers on Economic Activity*, 2000, no. 1, 1-60.

Alogoskoufis, G., Smith, R., 1991. The Phillips Curve, The persistence of Inflation, and the Lucas Critique: Evidence from Exchange Rate Regimes, *American Economic Review*, 81, no. 5, Dec., 1254-1275.

Bai J, Perron P., 1998. Estimating and testing linear models with multiple structural changes. *Econometrica* 66: 47-78.

Bai J, Perron P., 2003a. Critical values for multiple structural change tests, *Econometrics Journal*, 6, 72-78.

Bai J, Perron P., 2003b. Computation and analysis of multiple structural change models, *Journal of Applied Econometrics*, 18, 1-22.

Ball, L., 2000, Near Rationality and Inflation in Two Monetary Regimes, NBER Working Paper, no. 7988.

Baltagi, B.H., 2001. *Econometric Analysis of Panel Data*, second ed. New York: John Wiley and Sons.

Banerjee, B., Urga, G., 2005. Modelling structural breaks, long memory and stock market volatility: an overview, *Journal of Econometrics*, 129, 1-34.

Batini, N., Jackson, B., Nickell S., 2000. Inflation Dynamics and the Labour Share in the UK, Bank of England External MPC Unit Discussion Paper: No. 2 November.

Batini, N., Jackson, B., Nickell S., 2005. An Open-Economy New Keynesian Phillips Curve for the U.K., *Journal of Monetary Economics*, 52, 1061-71.

Box, G.E.P., Jenkins, G.M., 1976. *Time Series Analysis, Forecasting and Control*, San

Francisco, Holden Day.

Brainard, W.C., Perry, G.L., 2000. Making Policy in a Changing World, pp. 43-68 in Tobin, J., Perry, G., (Eds), *Economic Events, Ideas, and Policies: The 1960s and After*, Brookings.

Chen, Y-F., Russell. B., 2002. An Optimising Model of Price Adjustment with Missing Information, European University Institute Working Paper, ECO No. 2002/3.

Clarida, R., Gali, J., Gertler, M., 1999. The Science of Monetary Policy: a New Keynesian Perspective, *Journal of Economic Literature*, 37, 1661-1707.

Cogley, T., Sbordone ,A.M., 2005, A Search for a Structural Phillips Curve, Federal Reserve Bank of New York Staff Reports, no. 203.

Cogley, T., Sbordone ,A.M., 2006, Trend Inflation and Inflation Persistence in the New Keynesian Phillips Curve, Federal Reserve Bank of New York Staff Reports, no. 270.

Engle, R.F., Granger, C.W.J., 1987. Co-integration and Error Correction: Representation, Estimation, and Testing, Econometrica, 55, no. 2, 251-76.

Friedman, M., 1968. The role of monetary policy, *American Economic Review*, 58, 1-17.

Friedman, M., 1977. Nobel Lecture: Inflation and Unemployment, *The Journal of Political Economy*, 85, 451-72.

Gali, J., 2008. *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework:* Princeton University Press, Princeton.

Gali, J., Gertler, M., 1999. Inflation Dynamics: A Structural Econometric Analysis, *Journal of Monetary Economics*, 44, 195-222.

Gali, J., Gertler M., Lopez-Salido, J.D., 2001. European Inflation Dynamics, *European Economic Review*, 45, 1237-1270.

Gali, J., Gertler M., Lopez-Salido, J.D., 2005. Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve, *Journal of Monetary Economics*, 52, 1107-18.

Gordon, R.J., 1970. The Recent Acceleration of Inflation and its Lessons for the Future, *Brookings Papers on Economic Activity*, 1:1, 8-41.

Gordon, R.J., 1975. The Impact of Aggregate Demand on Prices, *Brookings Papers on Economic Activity*, 1:1, 613-62.

Gordon, R.J., 1977. Can the Inflation of the 1970s be Explained? *Brookings Papers on Economic Activity*, 6:3, 613-62.

Gordon, R.J., 1997. Time-varying NAIRU and its implications for Economic Policy, *Journal of Economic Perspectives*, 11, 11-32.

Granger, C.W.J., Newbold, P., 1974. Spurious Regressions in Econometrics, *Journal of Econometrics*, 2, 111-120.

Granger, C.W.J., Newbold, P., 1977. *Forecasting economic time series,* Academic Press, New York.

Hendry, D.F., 1980. Econometrics: Alchemy or Science?, *Econometrica*, 47, 387-406.

Henry, S.G.B., Pagan, A.R., 2004. The Econometrics of the New Keynesian Policy Model: Introduction, *Oxford Bulletin of Economics and Statistics*, 66, supplement, 581-607.

Hsiao, C., 2003. *Analysis of Panel Data*, second edition, Cambridge University Press, New York.

Ireland, P.N., 2007, Changes in the Federal Reserve's Inflation Target: Causes and Consequences, *Journal of Money, Credit and Banking*, 39, 1851-82.

Kiley, M.T., 2007, A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting, *Journal of Money, Credit and Banking*, supplement to 39, no. 1, 101-25.

King, R.G., Watson, M.W., 1994. The post-war U.S. Phillips curve: a revisionist econometric history, *Carnegie-Rochester Conference Series on Public Policy,* 41, 157-219.

Levin, A.T., Piger, J.M., 2002, Is Inflation Persistence Intrinsic In Industrial Economies?, Federal Reserve Bank of St. Louis Working Paper Series, no. 2002-023E.

McCallum, B.T., 1976. Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates, *Econometrica*, 44, 43-52.

Perron, P., 1989. The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis, *Econometrica*, 57, 1361-1401.

Perron, P., 1990. Testing for a Unit Root in a Time Series Regression with a Changing Mean, *Journal of Business and Economic Statistics*, 8, 153-162.

Perron, P., 1998. Further Evidence on Breaking Trend Functions in Macroeconomic Variables. *Journal of Econometrics*, 80, 355-385.

Phelps, E.S., 1967. Phillips curves, expectations of inflation, and optimal unemployment over time, *Economica*, 34, 254-81.

Phillips, A.W., 1958. The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957, *Economica*, 25, 1-17.

Phillips, P.C.B., 1986. Understanding spurious regressions in econometrics, *Journal of Econometrics*, 33, 311-40.

Plosser, C.I., Schwert, G.W., 1978. Money income and sunspots: Measuring economic relationships and the effects of differencing, *Journal of Monetary Economics*, 4, 637-60.

Popper, K.R., 1959. The Logic of Scientific Discovery, London, Hutchinson. Originally published as *Logik der Forschung* in 1934.

Roberts, J.M., 1995. New Keynesian Economics and the Phillips Curve, *Journal of Money, Credit and Banking*, vol. 27, 975-84.

Ross, S.A., Wachter, M.L., 1973. Wage Determination, Inflation, and the Industrial Structure, *American Economic Review*, 63, 675-92.

Rudd, J., Whelan, K., 2005. New Tests of the New-Keynesian Phillips Curve, *Journal of Monetary Economics*, 52, 1167-81.

Rudd, J., Whelan, K., 2006, Can Rational Expectations Sticky-Price Models Explain Inflation Dynamics?, *American Economic Review*, 96, 303-320.

Rudd, J., Whelan, K., 2007, Modeling Inflation Dynamics: A Critical Review of Recent Research, *Journal of Money, Credit and Banking*, supplement to 39, 155-170.

Russell, B., 1998. A Rules Based Model of Disequilibrium Price Adjustment with Missing Information, Dundee Discussion Papers, Department of Economic Studies, University of Dundee, November, No. 91.

Russell, B., 2006. Non-Stationary Inflation and the Markup: an Overview of the Research and some Implications for Policy, Dundee Discussion Papers, Department of Economic Studies, University of Dundee, August, No. 191.

Russell, B., Banerjee, A., 2008. The Long-run Phillips Curve and Non-stationary Inflation, *Journal of Macroeconomics*, 30, 1792-1815.

Russell, B., Evans, J., Preston, B., 2002. The Impact of Inflation and Uncertainty on the Optimum Markup set by Firms. European University Institute Working Paper, ECO No. 2002/2.

Smith, R.P., Fuertes, A-M, 2003. Panel Time Series, Cemmap working paper, Institute for Fiscal Studies, May.

Stock J.H., Watson, M.W., 2007, Why Has U.S. Inflation Become Harder to Forecast?, *Journal of Money, Credit and Banking*, supplement to 39, 3-33.

Sumner, M.T., Ward, R., 1983. The Reappearing Phillips Curve, *Oxford Economic Papers* supplement on The Causes of Unemployment, new series, 35, 306-20.

Svennson, L.E.O., 2000. Open Economy Inflation Targeting, *Journal of International Economics*, 50, 155-83.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton.

Yule, G.U., 1926. Why do we Sometimes Get Nonsense Correlations Between Time Series? A Study in Sampling and the Nature of Time Series, *Journal of the Royal Statistical Society*, 89, 1-64.

## DATA APPENDIX

The consumer price index (CPI) and unemployment rate data are seasonally adjusted and obtained directly from the United States of America, Bureau of Labour Studies (BLS). The monthly data for the period January 1952 to November 2004 was downloaded on 25 November 2004. The quarterly data for March 1952 to September 2004 is the average of the monthly data. The mnemonics are those from the BLS database. Gross domestic product (GDP) data was downloaded on 10 October 2010 from the United States of America, Bureau of Economic Analysis. All data are available at www.BillRussell.info.

---

**Table A1: Sources and details of the data manipulation**

---

*CPI inflation:* The monthly CPI is the United States city average, all items, 1982-84=100, ID: CUSSR0000SA0. CPI inflation is the change in the natural logarithm of the quarterly CPI multiplied by 400 to give the annualised rate.

*Unemployment rate*: The unemployment rate is the number of people over 16 years of age as a percentage of the non-institutionalised civilian population, ID: LNS14000000. The unemployment rate appears to have an increasing linear trend up to the middle to late 1970s and then a slight declining linear trend thereafter. Perron 1998 test confirms this and identifies a shift in the constant and break in trend in June 1978. The de-trended unemployment rate, $\left(U - U^*\right)_t$, is obtained by regressing the unemployment rate on a constant, a 'shift' dummy for June 1978 to September 2004, trend, a truncated trend that is zero up to and including March 1978 and then increasing in unit steps between June 1978 and September 2004, and a 'spike' dummy for June 1978.

*Gross domestic product:* Constant price GDP chained 2005 prices from line 1 of the NIPA Table 1.1.6. The data are de-trended with (i) a Hoddrick-Prescott filter, and (ii) an unrestricted regime based trend and constant for each of the eight regimes.

---

**Table 1: Estimated United States Inflation 'Regimes' using the Bai-Perion Technique**

| | *Variable* | *Dates of the 'Inflation Regimes'* | *Estimate* | *Standard Error* | *t-Value* |
|---|---|---|---|---|---|
| 1 | $\gamma_1$ | March 1952 to September 1955 | 0.001040 | 0.000894 | 1.2 |
| 2 | $\gamma_2$ | December 1955 to December 1966 | 0.004687 | 0.000533 | 8.8 |
| 3 | $\gamma_3$ | March 1967 to September 1972 | 0.011323 | 0.000746 | 15.2 |
| 4 | $\gamma_4$ | December 1972 to March 1975 | 0.023194 | 0.001131 | 20.5 |
| 5 | $\gamma_5$ | June 1975 to September 1977 | 0.014975 | 0.001131 | 13.2 |
| 6 | $\gamma_6$ | December 1977 to March 1981 | 0.027216 | 0.000956 | 28.5 |
| 7 | $\gamma_7$ | June 1981 to June 1990 | 0.010038 | 0.000588 | 17.1 |
| 8 | $\gamma_8$ | September 1990 to September 2004 | 0.006329 | 0.000482 | 13.1 |

Notes: Inflation is measured as the change in the natural logarithm of the United States CPI. In the graphs and the estimation this measure of inflation is multiplied by 400 to provide an 'annualised' rate of inflation.

## Table 2: Demonstrating the Standard Estimates are Biased

| | Dependent Variable: Actual Inflation $\Delta p_t$ | |
|---|---|---|
| **Independent Variable** ⇓ | **Hybrid** **1** | **Friedman-Phelps** **2** |
| $z_{t+1}$ | 1.2751 (4.8) | |
| $z_{t-1}$ | - 0.0927 (- 0.4) | 0.2703 (5.5) |
| $z_{t-2}$ | | 0.3139 (6.9) |
| $z_{t-3}$ | | 0.3151 (6.6) |
| $\left(U - U^*\right)_t$ | - 0.2874 (- 1.7) | - 0.4602 (- 4.1) |
| Constant | - 0.6961 (- 1.3) | 0.3447 (1.4) |
| Sum of Dynamic Inflation Terms | 1.1824 {0.1472} | 0.8993 {0.0630} |
| $\overline{R}^2$ | 0.07 | 0.71 |
| J-Test probability | [0.2961] | [0.4510] |
| LM1 test probability | [0.0000] | [0.0000] |
| LM4 test probability | [0.0000] | [0.0000] |
| ADF test residuals | - 5.4 | - 5.5 |

Notes: Standard errors reported as { }, *t*-statistics reported as ( ), and F-test probability values as [ ]. The models are estimated with 209 observations and re-estimated 10,000 times using Monte Carlo techniques estimated in WinRATS pro 6.2 with a 'seed value' of 171193. See equation (2) in the text for a description of how, $z_t$, is generated. The results reported are the means of the estimates from the Monte Carlo simulation of the model. Inference is unchanged if the median value of the estimates is reported instead of the mean value. The models are estimated by GMM with three lags of both inflation and the unemployment rate as instruments.

The J-test is the Hansen test for instrument validity. Rejection of the J-Test implies the instruments are invalid. LM1 and LM4 are Largrange Multiplier tests for first and fourth order serial correlation of the residuals respectively where the null hypothesis is no serial correlation. ADF test is the augmented Dickey-Fuller unit root test of the residuals where the 1 and 5 per cent critical values are – 2.576 and - 1.941 respectively..

**Table 3: 'Fixed Constant' Panel Estimates of United States Phillips Curves**

*Hybrid Model*

$$\Delta p_t^i = 0.5309 + 0.4882\,\Delta p_{t+1}^i + 0.4811\Delta p_{t-1}^i - 0.0704 U_t^i$$
$$(1.6) \quad (4.2) \qquad (4.3) \qquad (-1.1)$$

$\overline{R}^2$ = 0.85, Durban-Watson statistic 2.88. Hypothesis tests: $\Delta p_{t+1}^i + \Delta p_{t-1}^i = 0$ is rejected, $F_{(1,183)} = 922.5585$, p-val = 0.0000, and $\Delta p_{t+1}^i + \Delta p_{t-1}^i = 1$ is accepted $F_{(1,183)} = 0.9224$, p-val = 0.3381. Instruments: two lags of inflation and the unemployment rate.

*Friedman-Phelps Model*

$$\Delta p_t^i = 1.812 + 0.6280\Delta p_{t-1}^i - 0.0803\Delta p_{t-2}^i + 0.4233\Delta p_{t-3}^i - 0.1980 U_t^i$$
$$(2.7) \quad (8.1) \qquad (-0.9) \qquad (5.0) \qquad (-2.7)$$

$\overline{R}^2$ = 0.69, Durban-Watson statistic 2.03. Hypothesis tests: $\sum_{i=1}^{3}\Delta p_{t-i}^i = 0$, is rejected, $F_{(1,182)} = 287.1956$, p-val = 0.0000, and $\sum_{i=1}^{3}\Delta p_{t-i}^i = 1$ is accepted $F_{(1,182)} = 0.2568$, p-val = 0.6130. Instruments: three lags of inflation and two lags of the unemployment rate.

Notes: Phillips curve models are estimated with 187 usable observations in 8 cross-sections using EViews 5.1. Reported as ( ) are *t*-statistics. Models estimated with 2SLS and with the constant restricted to be the same across all 8 inflation regimes such that $\phi^1 = \phi^2 = ... = \phi^8$ in equation (5).

**Table 4: Fixed Effects Panel Estimates of United States Short-Run Phillips Curves**

| | Dependent Variable: Inflation $\Delta p_t^n$ | |
|---|---|---|
| | **Hybrid**<br>**1** | **Friedman-Phelps**<br>**2** |
| $\Delta p_{t+1}^n$ | 0.1376<br>(0.6) | |
| $\Delta p_{t-1}^n$ | 0.3360<br>(5.4) | 0.3323<br>(5.5) |
| $U_t^n$ | - 0.2643<br>(- 2.3) | -0.3159<br>(- 3.9) |
| Regime 1 | 1.2458<br>(2.1) | 1.5050<br>(3.1) |
| Regime 2 | 2.400<br>(2.5) | 2.9630<br>(6.1) |
| Regime 3 | 3.5974<br>(2.5) | 4.4935<br>(8.4) |
| Regime 4 | 7.0605<br>(2.5) | 8.6598<br>(10.0) |
| Regime 5 | 5.1537<br>(2.5) | 6.2960<br>(7.4) |
| Regime 6 | 7.6389<br>(2.4) | 9.6695<br>(10.7) |
| Regime 7 | 3.9048<br>(2.5) | 4.8105<br>(7.2) |
| Regime 8 | 2.8041<br>(2.5) | 3.34486<br>(6.7) |
| $\overline{R}^2$ | 0.75 | 0.60 |
| AR1<br>AR2<br>AR3<br>AR4 | [0.000]<br>[0.004]<br>[0.004]<br>[0.313] | [0.509]<br>[0.097]<br>[0.003]<br>[0.256] |
| *F-Tests* | | |
| $\phi_f + \phi_b = 0$ | [0.040] | [0.000] |
| $\phi_f + \phi_b = 1$ | [0.023] | [0.000] |
| F-statistic | [0.000] | [0.000] |
| $\phi^i = 0$ | [0.541] | [0.000] |

Notes: Reported as ( ) and [ ] are *t*-statistics and F-test probability values respectively. Estimated hybrid and F-P models have 8 cross-sections and 187 and 195 usable observations respectively. Instruments: two lags of inflation and the unemployment rate in both models. AR1 to AR4 are the Arellano-Bond tests of first to fourth order serial correlation in the residuals. F-statistic is testing that $\phi_f = \phi_b = \phi_U = \phi^i = 0$. Models estimated with 2SLS using Stata/SE 8.2 and Eviews 5.1. Tests of coefficient constancy: hybrid model, $\Delta p_{t+1}^i$, $F_{(7, 169)}$ = 0.75, [0.6306], $\Delta p_{t-1}^i$, $F_{(7, 169}$ = 0.26, [0.9686], $U_t^i$, $F_{(7, 169}$ = 0.61, [0.7496]. F-P model, $\Delta p_{t-1}^i$, $F_{(7, 178}$ = 0.36, [0.9252], $U_t^i$, $F_{(7, 178)}$ = 1.68, [0.1154]. Likelihood ratio omitted variable tests reject the inclusion of $\Delta p_{t-2}^i$, $F_{(1, 175)}$ = 0.7902, prob-value = 0.3753 and $U_{t-1}^i$, $F_{(1, 175)}$ = 0.9063, prob-value = 0.3424 in the hybrid model. Likelihood ratio omitted variable tests reject the inclusion of $\Delta p_{t-2}^i$, $F_{(1, 184)}$ = 0.0376, prob-value = 0.8465 and $U_{t-1}^i$, $F_{(1, 184)}$ = 1.0309, prob-value = 0.3113 in the F-P model.

**Table 5 Fixed Effects Panel Estimates of United States Short-Run Phillips Curves**

**De-trended Output and Unemployment Rate Forcing Variables**

| | **Dependent Variable: Inflation $\Delta p_t^n$** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **Hybrid** | | | | **Friedman-Phelps** | | | |
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| $\Delta p_{t+1}^n$ | 0.1260<br>(0.5) | 0.1954<br>(0.9) | - 0.0686<br>(-0.2) | 0.0679<br>(0.3) | | | | |
| $\Delta p_{t-1}^n$ | 0.3261<br>(5.2) | 0.3526<br>(5.4) | 0.3059<br>(4.8) | 0.3278<br>(5.3) | 0.3167<br>(5.3) | 0.3543<br>(5.8) | 0.2905<br>(4.9) | 0.3143<br>(5.4) |
| Constant | 2.0075<br>(2.2) | 1.6570<br>(2.3) | 2.7733<br>(2.5) | 2.2025<br>(2.3) | 2.5047<br>(10.5) | 2.3577<br>(9.7) | 2.5986<br>(11.0) | 2.5058<br>(10.8) |
| $\hat{y}_t^i$ | 17.1530<br>(1.8) | | | | 21.3013<br>(3.6) | | | |
| $yhp_t^i$ | | 16.6662<br>(1.7) | | | | 22.1509<br>(3.0) | | |
| $\hat{U}_t^i$ | | | - 0.5655<br>(- 2.6) | | | | - 0.5501<br>(- 4.5) | |
| $Uhp_t^i$ | | | | - 0.6427<br>(- 2.8) | | | | - 0.6772<br>(- 4.6) |
| $\overline{R}^2$ | 0.82 | 0.82 | 0.81 | 0.83 | 0.82 | 0.82 | 0.83 | 0.83 |
| AR1 | [0.000] | [0.000] | [0.038] | [0.001] | [0.669] | [0.298] | [0.984] | [0.417] |
| AR2 | [0.005] | [0.003] | [0.033] | [0.007] | [0.068] | [0.051] | [0.116] | [0.085] |
| AR3 | [0.009] | [0.005] | [0.010] | [0.007] | [0.009] | [0.006] | [0.003] | [0.003] |
| AR4 | [0.245] | [0.188] | [0.640] | [0.342] | [0.193] | [0.159] | [0.406] | [0.247] |
| *F-Tests* | | | | | | | | |
| $\phi_f + \phi_b = 0$ | [0.075] | [0.007] | [0.435] | [0.113] | [0.000] | [0.000] | [0.000] | [0.000] |
| $\phi_f + \phi_b = 1$ | [0.032] | [0.025] | [0.013] | [0.016] | [0.000] | [0.000] | [0.000] | [0.000] |
| F-statistic | [0.000] | [0.000] | [0.000] | [0.000] | [0.000] | [0.000] | [0.000] | [0.000] |
| $\phi^i = 0$ | [0.030] | [0.023] | [0.012] | [0.015] | [0.000] | [0.000] | [0.000] | [0.000] |

Notes: $y$ is constant price GDP. $U$ is the unemployment rate. The forcing variables are de-trended with an unrestricted trend and constant in each regime indicated by a '^' and with a Hoddrick-Prescott filter indicated by 'hp'. Models estimated with 2SLS using Stata 11.1 and Eviews 7.1. Constant reported in the table above is the weighted average of the constants i.e. the fixed effects from the eight cross-section panels. See also the notes to Table 4.

**Table 6: Estimates of the Implicit Long-run Phillips Curve**

*Linear*:

$$\overline{\Delta p} = -3.1112 + 1.3998\,\overline{U}, \quad R^2 = 0.19$$
$$(-2.0) \qquad (8.4)$$

The estimated coefficient on $\overline{U}$ is zero is rejected, $F_{(1,6)} = 71.9554$, prob-value = 0.0001. Standard error of the regression: 3.7062.

Non-linear Power Model

$$Ln\left(\overline{\Delta p}\right) = -4.8692 + 3.5043\, Ln\left(\overline{U}\right), \quad R^2 = 0.43$$
$$(-9.3) \qquad (14.9)$$

The estimated coefficient on $Ln\left(\overline{U}\right)$ is zero is rejected, $F_{(1,6)} = 221.1951$, prob-value = 0.0000. Standard error of the regression: 0.9103.

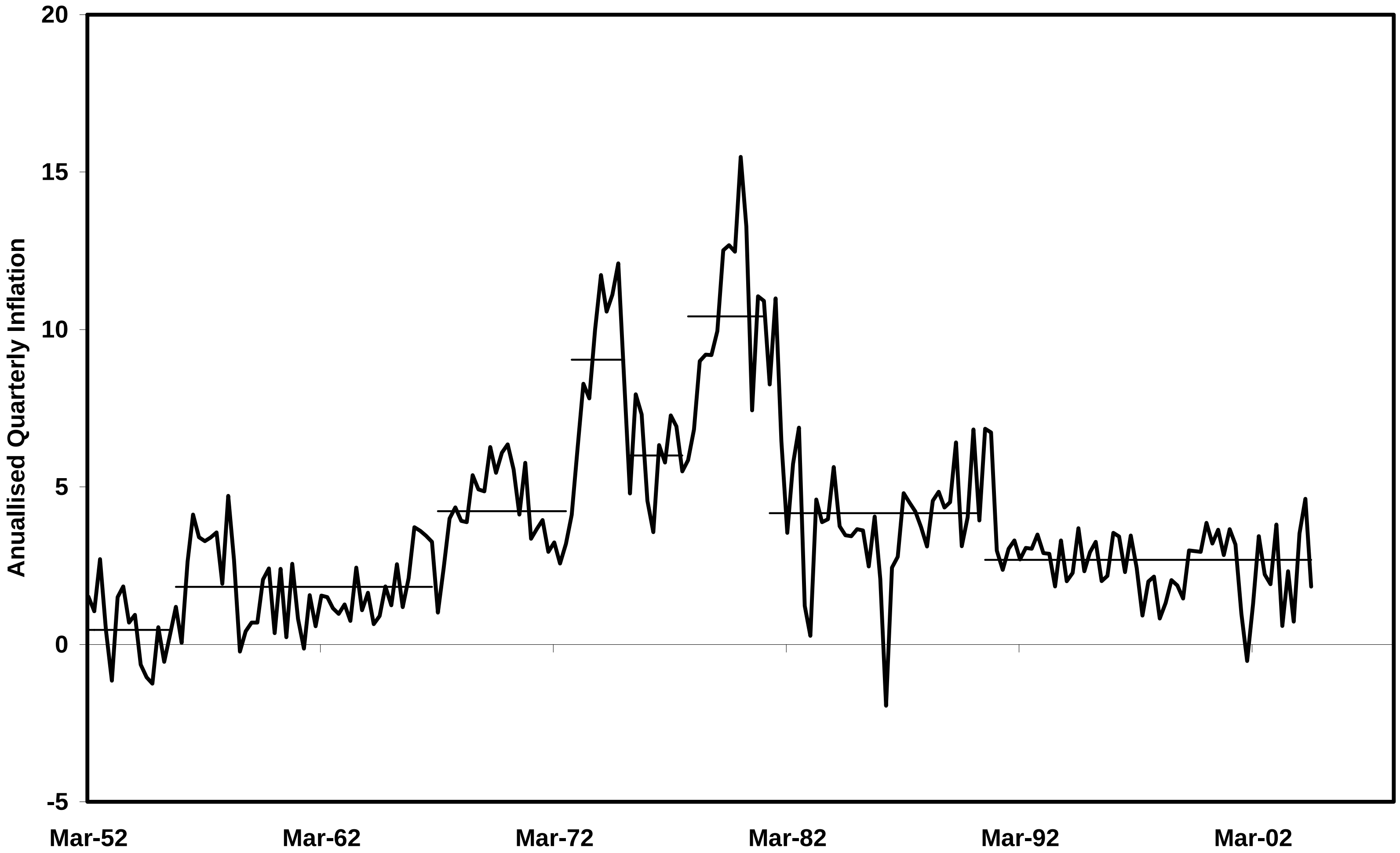
Non-linear Exponential Model

$$Ln\left(\overline{\Delta p}\right) = -\,2.0866 + 0.5708\,\overline{U}, \quad R^2 = 0.37$$
$$(-5.7) \qquad (10.9)$$

The estimated coefficient on $\overline{U}$ is zero is rejected, $F_{(1,6)} = 117.9987$, prob-value = 0.0000. Standard error of the regression: 0.9551.

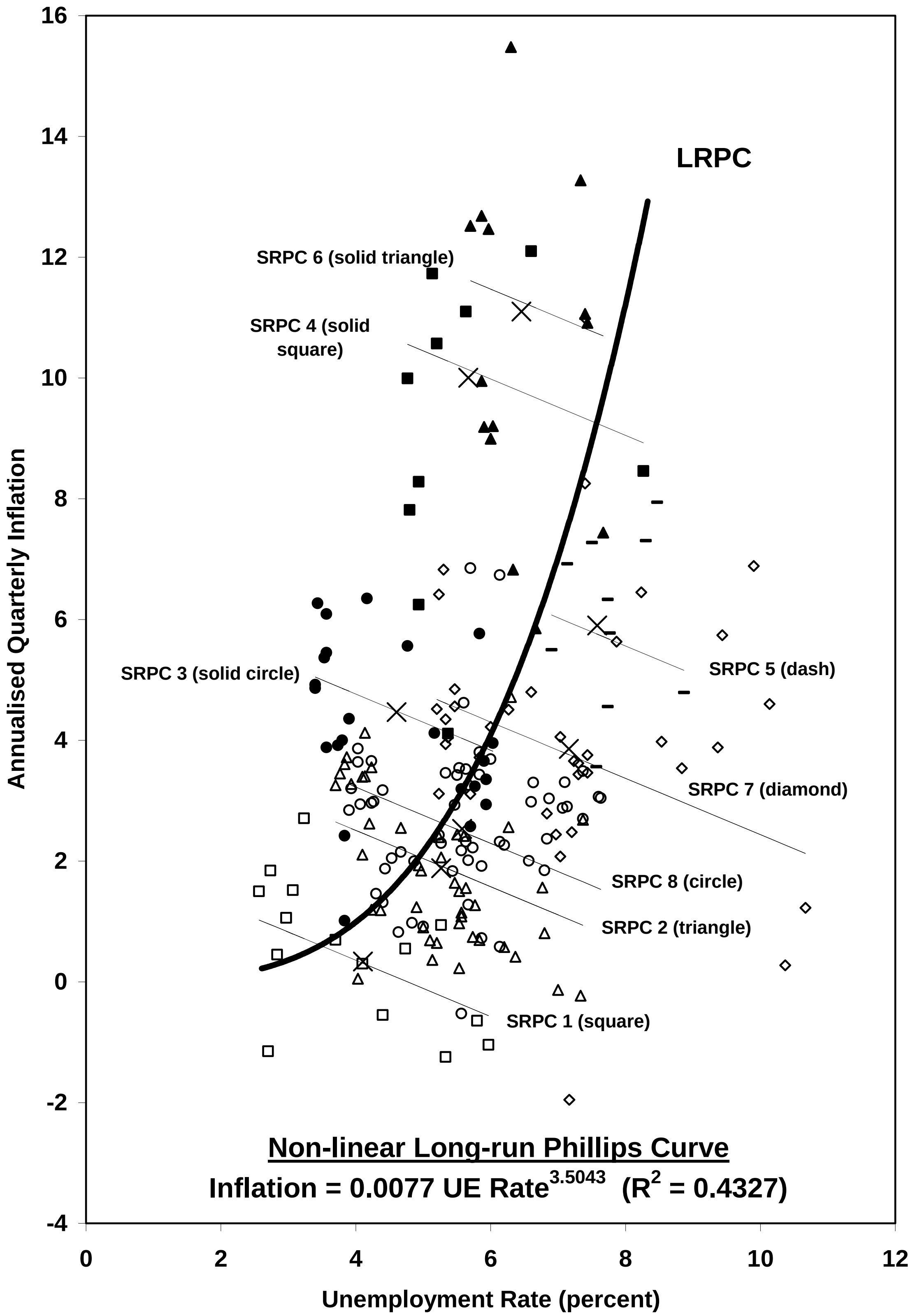
Notes: Models estimated with ordinary least squares and Newey-West HAC standard errors. Numbers in ( ) are *t* statistics. The calculation of the mean rates of inflation and unemployment exclude the first two observations in each regime to allow for the instruments and thereby conform to the estimates in Tables 4 and 5 and Graph 2. The estimates are unaffected in any material way if the mean is defined over all observations of the regime. Number of observations 8.

**Graph 1: United States Quarterly CPI Inflation, Seasonally Adjusted, March 1952 – September 2004**




Notes: Horizontal thin lines indicate the mean rates of inflation in the eight inflation regimes identified by the Bai-Perron technique. See the Data Appendix for details. Inflation is measured as the quarterly change in the natural logarithm of the CPI multiplied by 400.

**Graph 2: United States Phillips Curves**




Notes: LRPC is the 'power' non-linear curve reported in Table 6. Inflation is measured as the quarterly change in the natural logarithm of the CPI multiplied by 400. SRPC 1 to 8 are the short-run Phillips curves associated with regimes 1 to 8 reported in Table 1.