The Long-run Phillips Curve and Non-stationary Inflation[*]

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ABSTRACT

Modern theories of inflation incorporate a vertical long-run Phillips curve and are usually estimated using techniques that ignore the non-stationary behaviour of inflation. Consequently, the estimates obtained are imprecise and unable to test the veracity of a vertical long-run Phillips curve. We estimate a Phillips curve model taking into account the non-stationary properties in inflation and identify a small but significant positive relationship between inflation and unemployment. The results also provide some evidence that the trade-off between inflation and the rate of unemployment in the short-run worsens as the mean rate of inflation increases.

Keywords: Inflation, unemployment, long-run Phillips curve, business cycle, GMM.

JEL Classification: C22, C32, D40, E31

1. INTRODUCTION

The insight of Friedman (1968) and Phelps (1967) that permanently higher inflation would not lead to a permanent reduction in the unemployment rate and that the long-run Phillips curve is vertical underpins the modern theoretical, empirical and policy literature on inflation. Their insight is both simple and powerful. Simple in the sense that the concept of a vertical long-run Phillips curve now appears to be 'common sense'. And powerful in so far as this concept completely dominates modern macroeconomics.

One of the implications of a vertical long-run Phillips curve is that inflation may be non-stationary. Indeed, the argument that the original Phillips curve 'broke down' in the late 1960s and early 1970s implies that the expected rate of inflation had changed due to a change in the long-run rate of inflation. Therefore, the 'breakdown' was due to a period where inflation was non-stationary in the sense of having a time-varying mean. Subsequently, the issue of how rapidly expectations of inflation adjust to changes in the long-run rate of inflation came to be a crucial element of the debate surrounding adaptive and rational expectations.

To argue the converse, namely that inflation is stationary with a constant mean over the past five decades, would imply that (a) there has only been one short-run Phillips curve over this time (i.e. the curve where expected inflation matches the unique long-run rate of inflation); and (b) the long-run Phillips curve in a practical sense is a single point corresponding to a unique long-run rate of inflation and the long-run rate of unemployment. Unless we are willing to abandon all the Phillips curve literature since Friedman and Phelps that incorporates a vertical long-run Phillips curve, we must conclude that inflation is non-stationary.[1]

Recently it has again become popular to estimate Phillips curve models of inflation. For example see papers by Batini, Jackson and Nickell (2000, 2005), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001, 2005), and Rudd and Whelan (2005) who estimate the models using generalised method of moments (GMM) and instrumental variables to allow for

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[1] The argument that inflation is non-stationary, by considering the implications of the converse, is dealt with in more detail in Russell (2006, 2007).

correlation between the forward-looking variables, contemporaneous variables and the error term. It is well documented that this estimation approach is complicated by the problems surrounding the identification of the appropriate instruments and may lead to biased estimates and poor inference.[2]

A potentially greater problem is that the estimation technique itself will lead to biased estimates when inflation is an integrated process or is non-stationary due to shifting means. This bias is in addition to, and independent of, the biases introduced by the choice of instruments. In the following section we show that the behavioural emphasis placed on the estimates in terms of how expectations are formed and whether agents are forward- or backward-looking in these models is misplaced. This does not imply that agents do not behave in the ways outlined in the standard theories, only that empirical inflation models estimated using GMM provide such imprecise and biased estimates that they are unhelpful when trying to distinguish between the competing models. Furthermore, we conclude that these estimated models lack the precision necessary to identify whether or not the long-run Phillips curve is indeed vertical.

Graph 1 shows United States quarterly CPI inflation for the period March 1952 to September 2004.[3] The long period of low inflation in the 1950s and early 1960s is brought to an end by increasing inflation towards the end of the 1960s. This is followed by the high inflation of the 1970s and early 1980s associated with the Organisation of Petroleum Exporting Countries oil price increases and then two discrete reductions in inflation in the early 1980s and early 1990s. The first reduction is commonly referred to as the 'Volker deflation' and the second coincides with a large recession. These visual shifts in mean inflation can be shown more formally by a rolling 10-year regression of inflation on a constant in the form:

$$\Delta p_t = \gamma + \phi_t \tag{1}$$

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[2] See Pesaran (1981, 1987), Staiger and Stock (1997), Stock and Wright (2000), Stock, Wright, and Yogo (2002), Dufour (2003), Mavroeidis (2004, 2005), and Dufour, Khalaf and Kichian (2006).

[3] See the data appendix for details and sources of the data used in this paper.

where Δp_t is United States CPI inflation and γ is a constant. The estimated constant, $\hat{\gamma}$, along with the 5 per cent critical value are plotted in Graph 2 and shows the steady increase in the mean rate of inflation in the first half of the sample followed by a steady decline in the second half.

The changes in mean have not been confined to inflation. Graph 3 shows the United States unemployment rate for the period March 1952 to September 2004. We again see a general increase in the unemployment rate up until the early 1980s followed by a general decline thereafter. Similar to that in Graph 2 above, we can show this more formally by a 10-year rolling regression of the unemployment rate on a constant similar to that estimated in equation (1). The estimated constant from the 10-year rolling regression is shown in Graph 4.

How then can we examine Friedman and Phelps' original insight that the long-run Phillips curve is vertical? We argue below that, although the 'true' statistical process may be stationary with a very frequently shifting mean, we can approximate this process for our purposes as being integrated of order 1 (henceforth I(1)). One way forward, therefore, is to model inflation as an I(1) process which allows us to examine directly if there is a long-run relationship between inflation and the unemployment rate in the sense of Engle and Granger (1987), where unemployment is also taken to be I(1). In section 3 we estimate a two-variable I(1) system containing inflation and the unemployment rate. We find evidence for a small but significant *positive* slope to the long-run relationship between inflation and the unemployment rate. That is, persistently high inflation leads not only to unemployment returning to its long-run level but that the long-run unemployment rate is higher with higher inflation. Persistently high inflation leads to higher unemployment in the long-run.

All the 'modern' Phillips curve literatures since Friedman and Phelps incorporate a vertical Phillips curve if all the underlying assumptions are maintained. What slope the long-run Phillips curve might have if some of the assumptions are violated is not considered in these literatures. However, finding a small but significant positive long-run relationship between inflation and the rate of unemployment does not invalidate the underlying behavioural theories. Instead it simply implies that the inflation process is more complicated than that modelled in the modern Phillips curve theories since Friedman and Phelps.

Indeed Friedman (1977), in his Nobel lecture, conjectures that the long-run Phillips curve may have a positive slope. In this lecture he presents some graphical analysis based on five-year averaging of inflation and the unemployment rate to support this conjecture. We undertake a similar analysis by plotting in Graph 5 the rolling means of inflation against unemployment (taken from Graphs 2 and 4). This graph shows a clear positive correlation between mean unemployment and inflation. This suggests that Friedman's graphical analysis in 1977 near the peak in inflation continues to show in our data which includes the long general decline in inflation since the start of the 1980s.

One might argue that the positive long-run relationship that we identify between inflation and the rate of unemployment is simply due to supply-side influences on the long-run rate of unemployment. The steady increase in the rate of unemployment up until the early 1980s simply reflects a steady worsening in the supply-side influences on unemployment. Similarly, the steady decrease thereafter reflects a steady improvement in the supply-side influences which are independent of, and not associated with changes in the mean rate of inflation.

It is easy to understand that a positive supply shock will initially increase both inflation and the unemployment rate leading to a positive short-run relationship. However, in the longer term it is less understandable. A 'supply shock' to unemployment is likely to take a very long time to dissipate due to transaction costs, retraining, and the rigidity of human and physical capital. The same shock to inflation is likely to dissipate relatively quickly if firms are as sophisticated and rich in information as argued in the modern Phillips curve literature. Consequently, it is difficult for supply shocks to cause the *long-run* relationship between inflation and the rate of unemployment which requires both series to be highly persistent as is apparent in the data for developed economies.

Friedman (1977) argues that the positive slope to the long-run Phillips curve that he identifies may persist for many decades due to the greater uncertainty associated with higher inflation.[4]

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[4] Friedman (1977) argues, in what he calls the 'long-long-run', that the Phillips curve will again be vertical. However, in the transition period that may extend for many decades, the long-run Phillips curve will have a

This uncertainty persists in the transition due to the time taken for new arrangements to develop and deal with the higher uncertainty.

Our preferred explanation for the positive long-run relationship between inflation and the rate of unemployment is based on Banerjee and Russell (2001a, 2005) and Banerjee, Cockerell and Russell (2001). These papers identify a negative long-run relationship between inflation and the markup of price on unit labour costs for a range of developed economies including the United States. This relationship implies that there is a positive relationship between inflation and the real wage relative to productivity.[5] Consequently, persistently high inflation is associated with a persistently high real wage relative to the level of productivity. If high real wages relative to productivity leads to higher unemployment then the positive long-run relationship between inflation and the unemployment rate can be explained through the effect of inflation on the real wage. This explanation is examined in section 4 where we estimate a three-variable I(1) system consisting of inflation, the markup of price on unit labour costs and the rate of unemployment. Two long-run relationships are identified in the data. The first is the positive long-run relationship between inflation and the rate of unemployment and the second re-establishes the negative long-run relationship between inflation and the markup identified in our earlier work.

Our findings suggest that Friedman's (1977) conjecture was correct. It is a moot question whether the estimated relationships identified in the data between inflation and the rate of unemployment and between inflation and the markup are truly 'long-run' in the sense used by economic theorists.[6] It may be that after a number of decades (i.e. Friedman's 'long-long-

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positive slope. The slope persists due to the difficulty that economic agents have in fully adjusting to the new institutional and political arrangements that occur with changes in mean inflation.

5 The markup on unit labour costs, mu, can be written $mu = (p - w) + (y - l)$ where p, w, y, and l are the price level, wage rate, output and labour input respectively and lower case variables are in natural logarithms. Decreases in the markup, therefore, correspond to an increase in the real wage relative to the level of productivity.

6 The definition of the 'long run' for an economic theorist might include enough time so that all 'real' variables achieve their 'long-run' values when nominal variables are growing at a constant rate. Note that this definition of the 'long run' is model dependent as the long-run values of the real variables are defined in the model. Consequently, our empirical results might be rejected by an economic theorist as they imply the long-run Phillips curve is not vertical and so do not conform with modern Phillips curve models. However, to reject our results simply because they do not conform with the standard theoretical model is inconsistent with Popper's (1959, 1982) scientific method as we would be rejecting the empirical evidence rather than questioning the theoretical model itself.

run') of a constant high rate of inflation the relationship disappears. However, from a policy perspective the relationship appears to persist for enough time for it to be considered the 'long-run'. Certainly the persistence in the data easily satisfies the standard tests of the long-run in the sense of Engle and Granger (1987).

2. ESTIMATING THE HYBRID PHILLIPS CURVE

The mainstream literature that incorporates a vertical long-run Phillips curve can be understood in terms of the hybrid Phillips curve where inflation, Δp_t, in time t depends on expected inflation, Δp^e_{t+1}, conditioned on information available in time t, lagged inflation, Δp_{t-1}, and a 'forcing' variable, $u_t - u^*$.[7] The hybrid model can be written:

$$\Delta p_t = \delta_f \, \Delta p^e_{t+1} + \delta_b \, \Delta p_{t-1} + \delta_u \left(u_t - u^*\right) + \varepsilon_t \tag{2}$$

where the error term, ε_t, represents the random errors of agents and inflation shocks, Δ is the change in the variable and lower-case variables are in natural logarithms so that $\Delta p_t = p_t - p_{t-1}$. The 'forcing' variable represents excess demand and is measured in a variety of ways in the literature including the gap between real and potential output, real marginal costs, labour's share of income and, as shown here, the gap between the unemployment rate, u_t, and the long-run rate of unemployment, u^*.

The Friedman-Phelps Phillips curve and New Keynesian Phillips curve models are special cases of the hybrid Phillips curve. In the purely backward looking Freidman-Phelps model, agents hold adaptive expectations and so $\delta_f = 0$ and $\delta_b = 1$. In the purely forward-looking rational expectations New Keynesian models of Clarida, Galí and Gertler (1999) and Svensson (2000) $\delta_b = 0$ and $\delta_f = 1$. Finally, hybrid models assume that there are both forward and backward-looking price setting agents and that $\delta_f + \delta_b = 1$. The model provides a simple way to choose among the three competing standard theories of the Phillips

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[7] There are now a number of clear expositions of the New Keynesian Phillips curve and the hybrid Phillips curve. One such exposition is Henry and Pagan (2004).

curve by identifying the proportion of firms that base their pricing decisions on past inflation and expected inflation. The model also allows us to empirically verify the validity of the vertical long-run Phillips curve if we find that $\delta_f + \delta_b = 1$.

On a theoretical level, Phillips curve models that incorporate a vertical long-run Phillips curve make very strong statements concerning the statistical properties of the inflation data. If the model is true and $\delta_f + \delta_b = 1$ then inflation is non-stationary and empirical work should proceed by allowing for the possibility that inflation is non-stationary.

On a practical level, therefore, if we estimate Phillips curve models with non-stationary inflation data (either integrated or stationary with shifting means) using techniques such as GMM that are suitable for stationary data with a constant mean then the estimates will lack precision and are likely to be biased.[8] Consequently, any estimates of δ_f and δ_b that are retrieved will be unable to distinguish between the competing theories and whether or not $\delta_f + \delta_b = 1$. Very small deviations in these imprecise estimates will lead to very different conclusions in terms of choosing between the models and the slope of the long-run Phillips curve.

These observations concerning the estimation of hybrid Phillips curves can be demonstrated by estimating the model using GMM over the full sample between March 1952 and September 2004 and comparing the results with those obtained from a rolling 10-year estimation of the model. The results for the full sample are reported in Table 1. The last five columns provide critical values assuming that the data are stationary, CV^{S}_{prob}, a random walk, CV^{RW}_{prob}, and autoregressive with a coefficient on lagged inflation of 0.9, $CV^{0.9}_{prob}$. The degree of significance of the critical value is indicated by the subscript *prob*. We find that the coefficients on expected and lagged inflation are each significantly less than 1 while the sum

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[8] Estimating Phillips curve models that incorporate expected inflation and a contemporaneous 'forcing' variable imply that expected inflation will be correlated with the error term in equation (2). The standard solution is to estimate the model using GMM where the instruments are usually lags of both inflation and the 'forcing' variable. Considerable work has focused on overcoming problems when estimating hybrid Phillips curves using GMM. Most of the focus in the empirical literature is on determining whether or not lags in inflation and the 'forcing' variable are suitable instruments for the variables. As inflation more closely approximates an integrated variable the instruments become better predictors of expected inflation but simultaneously more correlated with the error term.

of the coefficients is insignificantly different from 1. Furthermore, forward-looking agents appear to have a larger, and possibly dominant, role in the inflation process. These results are similar to those found in the literature and can be compared with the results in the empirical Phillips curve papers listed in the introduction. The diagnostics indicate that the residuals are stationary but highly serially correlated and the *J*-test indicates that the instruments are not valid.

The poor model specification indicated by the diagnostic tests should not be overlooked. Estimating the hybrid Phillips curve on a rolling 10-year basis reveals the estimates of the model are poorly identified with the coefficients on expected and lagged inflation highly unstable. The three panels of Graph 6 report the rolling estimates of the coefficients on expected inflation, lagged inflation and the sum of the inflation coefficients. Also shown on the graphs are the critical values testing for the coefficients equal to 1 assuming that the data are stationary. We see that for much of the period the coefficients on expected and lagged inflation display large variation and that for much of the time they are significantly less than 1 based on critical values assuming that the data are stationary. We also see in the bottom panel that for nearly the entire sample the sum of the coefficients is insignificantly different from 1. However the imprecision of the estimates is evident in the graphs. We cannot reject that the sum of the coefficient is 1 even though the point estimates of the sum range between around zero and 2 ½.

These results draw attention to two issues. First, the estimates are very poorly determined with very large standard errors. Consequently, it is very difficult to reject hypotheses concerning the size of the estimates. We cannot be confident from these results that the long-run Phillips curve is vertical even though we cannot reject the hypothesis that the sum of the coefficients equals to one. It may be that the long-run Phillips curve has a small but economically important slope.

Second, placing a behavioural emphasis on these results in terms of the expectation formation or proportions of agents who are forward or backward looking is misplaced as the estimates are so inaccurate that we cannot reject most hypotheses. Furthermore, given the importance of the behavioural hypotheses in defining their respective models the behaviour should be stable over time. If the behavioural arguments are correct then it appears from the variation in the estimates of δ_f and δ_b reported in Graph 6 that agents appear to forget quickly and learn again only to forget again.

3. The Long-run Inflation-Unemployment Rate Relationship

So far we have not addressed the issue of what type of non-stationarity is present in the inflation data. Within our framework, two forms of non-stationarity present themselves as interesting candidates for consideration. The first is to assume that monetary authorities respond to a series of shocks by making discrete changes in their implicit inflation target, leading inflation to be non-stationary with shifting means. The usual way to proceed here would be to introduce as many shift dummies as necessary to 'render' inflation a stationary series. Typically this will not require a large number of shift dummies. Invariably this small number of dummies will not identify all the shifts in the mean rate of inflation and the unidentified shifts will continue to be problematic when estimating inflation models. Furthermore, it is not easy to interpret the dummies in an economic sense other than they indicate shifts in the mean rate of inflation.

The second way to proceed is to assume that inflation shocks are very frequent and that monetary authorities adjust the target rate of inflation at least partially in response to the shocks. This assumption is consistent with the view that central banks did not actively offset the increases in inflation during the 1970s by increasing nominal interest rates by more than the increase in expected inflation (for example see McCullum 2000). This in turn suggests that central banks accommodated the higher inflation and raised their implicit inflation target in the 1970s. Furthermore, they may well have acted in a similar fashion when shocks served to reduce inflation at other times like in the early 1950s and 1990s. We therefore assume that, while the 'true' statistical process of inflation may be stationary around a very frequently shifting mean, this process can be approximated by an I(1) process.

The hybrid Phillips curve model in equation (2) suggests that the inflation process can be modelled in terms of four variables, namely inflation, expected inflation, the unemployment rate and the long-run unemployment rate. In the long run, expectations are realised and expected inflation equals actual inflation. Similarly, actual unemployment will equal the long-run unemployment rate. If we assume that the long-run rate of unemployment may not be fixed then it can vary in the long run. In this case, the long-run relationship that implicitly underpins the hybrid Phillips curve when expectations are realised can be written:

$$\Delta p_t = \alpha_0 + \alpha_1 u_t^* + \varepsilon_t \qquad (3)$$

In the standard models that incorporate a vertical long-run Phillips curve, the long-run coefficient α_1 is equal to zero. Alternatively, if $\alpha_1 \neq 0$ then the long-run Phillips curve is not vertical. The advantage of this approach over estimating the model using GMM is twofold. First, by estimating (3) we can allow for the possibility that the variables are non-stationary and in so doing improve the precision of the estimates. And second, estimating equation (3) allowing for non-stationary data allows us to either reject or accept the vertical long-run relationship. This is in contrast with the GMM estimates of the hybrid Phillips curve model which is unable to reject the vertical Phillips curve when the data are non-stationary as the estimates are too imprecise.

However, these advantages do come with a drawback insofar as estimating equation (3) does not allow us to distinguish among the three competing standard theories of inflation. In particular, the estimates cannot distinguish between the proportions of forward and backward looking agents or how expectations are formed. Estimating equation (3) allowing for non-stationary data simply allows us to answer the question of the slope of the long-run Phillips curve and provide better estimates of the dynamics of inflation.

If we assume that inflation is I(1), we can estimate a two-variable I(1) system containing inflation and the unemployment rate.[9] The system is conditioned on a predetermined business cycle variable which is measured as de-trended constant price gross domestic product (GDP) and a series of spike dummies for periods when the residuals of each estimated equation in the system are greater than 3 standard errors.[10]

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9 One advantage of this two-variable system is that it is directly comparable with much of the 'modern' Phillips curve literature that usually estimates single-equation Phillips curve models of inflation and a forcing variable where the latter is often the unemployment rate.

10 Excluding the dummies from the models does not affect (i) the estimate of the long-run relationship in a meaningful way; (ii) inference concerning the number of long-run relationships in the data; and (iii)

The results of estimating the inflation-unemployment rate I(1) system are provided in Table 2a. The L-max and Trace statistics for the number of cointegrating vectors indicate that the hypothesis of one long-run relationship between the variables is evident in the data. We find that the long-run coefficient, α_1, is significant and negative indicating a positive long-run relationship between inflation and the unemployment rate. The estimated long-run coefficient is – 2.714 and implies that a 1 percentage point increase in annual inflation is associated with approximately a 0.37 of a percentage point increase in the rate of unemployment in the long run. This estimate can be compared with Graphs 2 and 4 where we see that mean inflation increased by around 7 percentage points between the start of 1962 and the peak in 1982 while the unemployment rate increased by about 3 percentage points over the same period. A 'crude' estimate of the long-run relationship from these figures would suggest a long-run coefficient of around 0.4 (i.e. 3 divided by 7). The stability of the long-run estimate is demonstrated in Graph 7 which shows the max test of constancy, Q(t), of the cointegration vector (as described in the CATS 2.0 manual) is considerably less than the 95 per cent critical value of 1. This test compares the full sample estimate of the cointegration vector with estimates derived from recursive estimation of the model and constructs a statistic based on the maximum difference between these estimates.

3.1 The Short-run Dynamics of the Model

Table 2b provides estimates of the model's dynamics indicating that the business cycle has a large negative effect on the unemployment rate but that its direct impact on inflation is not significant. It also appears that inflation is weakly exogenous given that the long-run relationship is not significant in the inflation equation.

Unfortunately the dynamics of the short-run Phillips curves is not derivable from this I(1) analysis as each short-run curve is conditional on a given mean, or long-run, rate of inflation which cannot be identified. However, the interaction of the short-run dynamics with the long-run relationship can be examined through impulse response analysis on the estimated two-variable system.

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stability of the long-run estimates. However, the inclusion of these dummies improves the dynamic specification of the models.

The analysis is in two parts. The first is a one standard deviation shock to the unemployment rate which is shown in the two left-hand panels of Graph 8. In the initial periods following the shock the familiar negative short-run Phillips curve is evident with the higher unemployment rate associated with lower inflation. We could interpret this shock as a demand shock to the system. However, as we travel further from the shock the long-run positive relationship between inflation and the unemployment rate asserts itself on the unemployment rate and after 20 quarters unemployment is eventually lower. This is because the inflation rate is permanently lower following the initial shock to the rate of unemployment. Note that after 20 quarters the ratio of inflation to unemployment is around 3.188 (i.e. - 0.102/ - 0.032) which can be compared with the long-run relationship of 2.714.

The second part of the analysis is a one standard deviation shock to inflation which may be interpreted as a 'supply shock' to the system and is shown in the two right-hand panels of Graph 8. We see that in the short-run and in the long-run the higher inflation is associated with a higher unemployment rate. Note again that after 20 quarters the ratio of inflation to the unemployment rate is 2.752 (i.e. 0.622 / 0.226) compared with the long-run relationship of 2.714.

3.2 A Graphical Representation of the Results

The solid thick line on Graph 9, labelled LR, is the estimated long-run Phillips curve with the actual combinations of inflation and the unemployment rate used in the estimation shown by the scattering of symbols. The positive relationship is evident in the data and the long-run relationship.

The actual data on the graph are separated into 5 inflationary 'episodes' that are listed below the graph along with the identifying symbols and mean rates of inflation. We would expect the actual inflation and unemployment rate data to be strung out and clustered around a series of short-run Phillips curves associated with each inflationary episode. Unfortunately the short-run Phillips curves cannot be identified directly from this analysis. As a proxy for these short-run Phillips curves, the trend lines for the data in each inflationary episode are shown as thin lines and identified with numbers 1 to 5 that correspond to each inflationary episode in the table below the graph.

If we consider the data in each episode we see that at the start of the sample we are in a period of low inflation with low unemployment rates (diamonds). This is followed by a period of increasing inflation and unemployment rates at the end of the 1960s and early 1970s (triangles) and then the high inflation and high unemployment of the 1970s (crosses). The deflation of the 1980s (squares) and the 1990s (dashes) see us again return to a low inflation and low unemployment environment on the graph. What is made clear by separating the data into inflationary episodes is that as inflation rises the short-run Phillips curves as proxied by the thin trend lines and the associated cluster of data not only moves upwards but also to the right. Importantly, this process is reversed with declining inflationary episodes and we see the short-run Phillips curve shifting not only down as expected but to the left. It is the apparent sideway shifts, rather than purely vertical shifts, of the short-run curve that justifies our result of a positive sloping long-run Phillips curve.

4. EXPLAINING THE LONG-RUN INFLATION-UNEMPLOYMENT RATE RELATIONSHIP

An explanation for the positive slope of the long-run Phillips curve is provided in a series of papers by us that estimate a negative long-run relationship between inflation and the markup of price on unit costs for a number of developed economies.[11] This suggests that higher inflation leads to a lower markup of price on unit costs in the long-run or, equivalently, a higher real wage relative to the level of productivity. If high real wages relative to productivity increase unemployment then high inflation will be associated with high unemployment and lead to a positively sloping long-run Phillips curve.

As we have discussed briefly in Banerjee and Russell (2005) and in more detail in Russell (2006), explanations of the negative long-run inflation markup relationship can be separated into three groups. The first focuses on the interaction of inflation with competition. Bénabou (1988, 1992) and Diamond (1993) argue that higher inflation and the associated greater dispersion of prices increases the returns to search and, therefore, the quantity of search by customers also increases. In this case higher inflation leads to greater competition and a lower markup. The second group makes use of the ‘menu’ cost arguments of Mankiw

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[11] For example, see the estimates of the long-run inflation-markup relationship provided in Banerjee, Cockerell and Russell (2001), and Banerjee and Russell (2001a, 2001b, 2004, 2005).

(1985) and Parkin (1986) where firms face small costs when adjusting prices.[12] Although the costs are small, they lead to non-negligible welfare effects and firms find that they lack the incentive to adjust back to general equilibrium prices. The difficulty with these two groups of explanations is that for the inflation-markup relationship to persist in the long run, competition must be permanently higher with higher inflation in the first group of explanations and the price rule must be invariant to higher inflation in the second.

The third group of explanations focus on the difficulties that non-colluding price-setting firms face when adjusting prices in an economic environment where information is missing. Russell, Evans and Preston (2002) argue that marginal costs are difficult to allocate to jointly produced output. Consequently the profit maximising price is not known with certainty. Furthermore, firms believe they face an asymmetric loss function where setting too high a price relative to the profit maximising price costs the firm more in lost profits than setting too low a price. Therefore firms act cautiously in this economic environment and set a price lower than the profit maximising price. Finally, if uncertainty increases with inflation, firms act more cautiously with higher inflation and set even lower prices. Russell (1998) and Chen and Russell (2002) also assume that firms set prices and do not collude. The former argues that in an uncertain environment firms follow a rule when adjusting prices so as to coordinate price changes between firms. The later argues that firms try to minimise the loss to the firm when adjusting prices in an inflationary environment. In these three papers the lower markup associated with higher inflation is interpreted as the cost to firms of overcoming the missing information in the model. Importantly these explanations explicitly argue that inflation and the markup are negatively related in the steady state and this is interpreted as a long-run relationship.

To examine the proposition that the positive slope to the long-run Phillips curve may be due to a negative long-run relationship between inflation and the markup, we estimate a three variable I(1) system. The system includes inflation, the markup of price on unit labour costs, and the rate of unemployment. We expect that the data will show two long-run relationships. The first is that identified in the previous section between inflation and the rate of

[12] For example see, Rotemberg (1983), Kuran (1986), Naish (1986), Danziger (1988), Konieczny (1990) and Bénabou and Konieczny (1994).

unemployment. The second relationship follows Banerjee, Cockerell and Russell (2001) and is given by:[13]

$$\Delta p = \beta_0 + \beta_1 \; mu \qquad (4)$$

where mu is the markup of price on unit labour costs and β_1 is the parameter that measures the trade-off in the long-run between inflation and the markup. The markup is calculated as $p - ulc$, where the price level, p, is the consumer price index and ulc is a measure of unit labour costs. The long-run model is again conditioned on a business cycle variable and a series of spike dummies that capture the sometimes erratic behaviour of the data.

In the standard Phillips curve models, inflation has no impact on the markup in the long run since the real wage returns to its constant long-run value on the vertical long-run Phillips curve. Thus $\beta_1 = 0$ in equation (4) and $\alpha_1 = 0$ in equation (3). In our more general model estimated here we expect that $\beta_1 < 0$ as in earlier work and $\alpha_1 > 0$ as estimated in section 3.

The results of estimating the three-variable, inflation-unemployment rate-markup I(1) system are provided in Tables 3a and 3b. The L-max and Trace statistic indicate that we can comfortably accept the hypothesis of two long-run relationships in the data and these are reported at the top of Table 3a. The first long-run relationship between inflation and the unemployment rate is slightly larger but similar in magnitude to that reported above with a long-run coefficient of – 3.271. The second long-run relationship between inflation and the markup reports a long-run coefficient of 26.344 which is slightly larger but also similar to estimates reported in earlier work.[14] Both long-run relationships are highly significantly different from zero and the lower panel of Graph 7 demonstrates that the two long-run relationships are stable over the past five decades considered in this analysis.

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[13] The two long-run relationships of equations (3) and (4) cannot strictly be true since the markup approaches zero and the unemployment rate approaches infinity as inflation tends to infinity. This suggests the 'true' relationships are non-linear. However, over the small range of inflation as experienced in the United States over the last 50 years, the log-linear relationship appears to be a good approximation.

[14] The long-run inflation-markup coefficient can be re-parameterised and compared with earlier work. The re-parameterised value is 3.795 (i.e. 100 divided by 26.344). Earlier estimates for the United States reported in Banerjee and Russell (2001a, 2001b) of 0.62 using annual 'private sector' gross domestic

We can use the results of Table 3a to show that the positive slope of the long-run Phillips curve may be explained by the negative inflation-markup long-run relationship. Imposing an alternative set of normalising and exclusion restrictions to identify the long-run relationships leads to an equivalent representation of the cointegrating space to that reported in Table 3a. In this case we retrieve the following long-run relationships:

$$LR1: \ \Delta p = \beta_0 - 26.344\ mu \qquad (5)$$

$$LR2: \ \ u = \chi_0 - 8.055\ mu \qquad (6)$$

The first long-run relationship, *LR*1, is again the inflation-markup relationship. The second, *LR*2, is a long-run relationship between the unemployment rate and the markup which is consistent with our assertion above that higher real wages relative to productivity (i.e. a lower markup) is associated with higher unemployment rates. Finally, note in Table 3b that inflation is weakly exogenous in this three variable I(1) system. One might thus characterise the results as saying that higher inflation in the 1970s and early 1980s was associated with a lower markup which in turn meant a higher real wage. And that the higher real wage was associated with higher unemployment.

5. CONCLUSION

This paper argues that modern theoretical models of inflation suggest that inflation is non-stationary. However, empirical work on identifying which of the competing theories 'best' describes the inflationary process appears to ignore resolutely the possibility that the inflation data may be non-stationary. Consequently, inflation models are typically estimated using techniques that are best suited for stationary data that have a constant mean. The resulting estimates are likely to be biased and lack precision so that inference concerning which model is the 'best' is poor. The problem of poorly estimated coefficients makes it impossible to test properly whether or not the long-run Phillips curve is vertical as small deviations in the estimated inflation coefficients from one imply that the long-run curve has a non-zero slope.

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product data and 0.46 using aggregate gross domestic product data and 6.602 using a measure of marginal cost markups and 2.526 using unit cost markups in Banerjee and Russell (2005).

To overcome this problem we explicitly acknowledge the non-stationary nature of inflation in our estimation of the long-run Phillips curve. We find that there is a small but significant positive slope to the long-run Phillips curve. This slope implies that a 5 percentage point increase in the general rate of inflation leads to an increase in the unemployment rate in the long-run of around 1 ½ percentage points. Movements of this magnitude in the unemployment rate are likely to be important in economic, social and political senses.

Finally, notice on Graph 9 that the proxies for the short-run Phillips curves become steeper with increases in the mean rates of inflation. The trend lines for the low inflation episodes numbered 1 and 5 are almost horizontal. As the mean rates of inflation increase the trend lines rotate in a clockwise direction and become steeper suggesting the short-run trade-off between inflation and unemployment diminishes. The very flat short-run Phillips curves at low mean rates of inflation suggests that the temptation for governments to use expansionary macroeconomic policy is great. However, the positive slope to the long-run Phillips curves argues strongly against such myopic behaviour and lends support to the idea of independent central banks that deliver low stable inflation.

A1 DATA APPENDIX

The CPI data and unemployment rate data were downloaded from the United States of America, Bureau of Labour Studies. The national accounts data were downloaded from the National Income and Product Account tables from the United States of America, Bureau of Economic Analysis. The data for March 1952 to September 2004 was downloaded on 25 November 2004. Except where indicated, the data are quarterly and seasonally adjusted. The mnemonics are from the databases that they were downloaded from.

Table A1: Sources and details of the data manipulation

Variable	*Details*
Consumer price inflation	The monthly CPI is the US city average, all items, 1982-84=100, ID: CUSSR0000SA0. The derived quarterly data is the average of the monthly data. CPI inflation is the change in the natural logarithm of the quarterly CPI multiplied by 400 to give the annualised rate.
Unemployment rate	The unemployment rate is the number of people over 16 years as a percentage of the non-institutionalised civilian population, ID: LNS14000000. The derived quarterly data is the average of the monthly data.
Gross domestic product (GDP) at constant prices	Constant price GDP at 2000 prices, Table 1.1.6, line 1.
Gross domestic product (GDP) implicit price deflator at factor cost	Nominal GDP at factor cost is nominal GDP (Table 1.1.5, line 2) plus subsidies (Table 1.10, line 10) less taxes (Table 1.10, line 9). GDP implicit price deflator is nominal GDP at factor cost divided by constant price GDP.
Unit labour costs	Calculated as total labour compensation divided by constant price GDP where the former is wages, salaries and supplements, Table 1.10, line 2.
Markup	Calculated as the consumer price index divided by unit labour costs.
De-trended Variables	The variables are de-trended using broken linear trends. The breaks are identified using the augmented Perron (1997) unit root test which allows for the presence of an endogenous change in the level and slope of the trend function. Having identified the break (or breaks by using the technique sequentially) the data is regressed on a constant and trend as well as a 'short' trend, shift constant, and spike dummy for each break in trend. The short trend is zero for each period up to the break and then a linear trend thereafter. The shift dummy is zero up until the break and one thereafter. The 95 % critical value is - 3.13. Unemployment Rate: There is a break in trend and constant in June 1978 (test statistic -4.1). Business cycle (measured as de-trended natural logarithm of constant price GDP): The test identifies two breaks in the trend and constant. September 1963 (test statistic – 4.8). December 1979 (test statistic – 5.6).

6. REFERENCES

Banerjee, A., Cockerell, L. and B. Russell (2001). An I(2) analysis of Inflation and the Markup, Journal of Applied Econometrics, Sargan Special Issue, vol. 16, pp. 221-240.

Banerjee, A. and B. Russell (2001a). The Relationship between the Markup and Inflation in the G7 Economies and Australia, Review of Economics and Statistics, vol. 83, No. 2, May, pp. 377-87.

Banerjee, A. and B. Russell. (2001b). Industry Structure and the Dynamics of Price Adjustment, Applied Economics, 33:17, pp. 1889-901.

Banerjee, A. and B. Russell (2004) A Reinvestigation of the Markup and the Business Cycle, Economic Modelling, vol. 21, pp. 267-84.

Banerjee, A. and B. Russell (2005). Inflation and Measures of the Markup, Journal of Macroeconomics, vol. 27, pp. 289-306.

Batini, N., B. Jackson, and S. Nickell. (2000). Inflation Dynamics and the Labour Share in the UK, Bank of England External MPC Unit Discussion Paper: No. 2 November.

Batini, N., B. Jackson, and S. Nickell. (2005). An Open-Economy New Keynesian Phillips Curve for the U.K., Journal of Monetary Economics, vol. 52, pp. 1061-71.

Bénabou, R. (1988). "Search, Price Setting and Inflation." Review of Economic Studies, 55:3 July, pp. 353-73.

Bénabou, R. (1992). "Inflation and Markups: Theories and Evidence from the Retail Trade Sector." European Economic Review, 36, pp. 566-74.

Bénabou, R. and J.D. Konieczny. (1994). "On Inflation and Output with Costly Price Changes: A Simple Unifying Result." American Economic Review, 84:1 March, pp. 290-7.

Cats in Rats Cointegration Analysis of Time Series, Version 2, Estima, Evanston, Illinois.

Chen, Y-F and B. Russell. (2002). "An Optimising Model of Price Adjustment with Missing Information." European University Institute Working Paper, ECO No. 2002/3.

Clarida, R., Gali, J. and Gertler, M. (1999). The Science of Monetary Policy: a New Keynesian Perspective, Journal of Economic Literature, vol. 37, pp. 1661-1707.

Danziger, L. (1988). "Costs of Price Adjustment and the Welfare Economics of Inflation and Disinflation." American Economic Review, 78:4 September, pp. 633-46.

Diamond, P. (1993). "Search, Sticky Prices and Inflation." Review of Economic Studies, 60:1 January, pp. 53-68.

Dufour, J-M. (2003). Identification, Weak Instruments and Statistical Inference in Econometrics, Canadian Journal of Economics, vol. 36 (4), pp. 767-808.

Dufour, J-M, Khalaf, L. and M. Kichian (2006). Inflation Dynamics and the New Keynesian Phillips Curve: An Identification Robust Econometric Analysis, Journal of Economic Dynamics & Control, vol. 30, pp. 1707-27.

Engle, R.F. and C.W.J. Granger, (1987). Co-integration and Error Correction: Representation, Estimation, and Testing, Econometrica, vol. 55, no. 2, May, pp. 251-76.

Friedman, M. (1968). The role of monetary policy, American Economic Review, 58, 1 (March), pp. 1-17.

Friedman, M. (1977). Nobel Lecture: Inflation and Unemployment, The Journal of Political Economy, vol. 85, no. 3, pp. 451-72.

Gali, J., and M. Gertler (1999). Inflation Dynamics: A Structural Econometric Analysis, Journal of Monetary Economics, vol. 44, pp. 195-222.

Gali, J., Gertler M., and J.D. Lopez-Salido (2001). European Inflation Dynamics, European Economic Review, vol. 45, pp. 1237-1270.

Gali, J., Gertler M., and J.D. Lopez-Salido (2005). Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve, Journal of Monetary Economics, vol. 52, pp. 1107-18.

Hansen, L.P. (1982). Large Sample Properties of Generalized Method of Moments Estimators, Econometrica, Vol. 50, no. 4, July, pp. 1029-54.

Henry, S.G.B. and A.R. Pagan (2004). The Econometrics of the New Keynesian Policy

Model: Introduction, Oxford Bulletin of Economics and Statistics, vol. 66, supplement, pp. 581-607.

Konieczny, J.D. (1990). "Inflation, Output and Labour Productivity when Prices are Changed Infrequently." Economica, 57:May, pp. 201-18.

Kuran, T. (1986). "Price Adjustment Costs, Anticipated Inflation, and Output." Quarterly Journal of Economics, 71:5 December, pp. 1020-7.

Mankiw, G.N. (1985). "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly." Quarterly Journal of Economics, 100:May, pp. 529-37.

Mavroeidis, S. (2004). Weak Identification of Forward-looking Models in Monetary Economics, Oxford Bulletin of Economics and Statistics, vol. 66, supplement, pp. 609-35.

Mavroeidis, S. (2005). Identification Issues in Forward-looking Models Estimated by GMM, with an Application to the Phillips Curve, Journal of Money, Credit and Banking, vol. 37, pp. 421-48.

McCullum, B.T. (2000). Alternative Monetary Policy Rules: A Comparison with Historical Settings for the United States, the United Kingdom, and Japan, Federal Reserve Bank of Richmond Economic Quarterly, vol. 86, Winter, pp. 49-79.

Naish, H.F. (1986). "Price Adjustment Costs and the Output-Inflation Trade-Off." Economica, 53, pp. 219-30.

Parkin, M. (1986). "The Output-Inflation Trade-off when Prices are Costly to Change." Journal of Political Economy, 94:February, pp. 200-24.

Perron, P., (1997). Further Evidence on Breaking Trend Functions in Macroeconomic Variables. Journal of Econometrics, 80 no. 2, 355-385.

Pesaran, M.H. (1981). Identification or Rational Expectations Models, Journal of Econometrics, vol. 16, no. 3, pp. 375-98.

Pesaran, M.H. (1987). The Limits to Rational Expectations, Blackwell, Oxford.

Phelps, E.S. (1967). Phillips curves, expectations of inflation, and optimal unemployment over time, Economica, 34, 3 (August), pp. 254-81.

Popper, K.R. (1959). The Logic of Scientific Discovery, London, Hutchinson.

Popper, K.R. (1982). Realism and the Aim of Science, London, Hutchinson.

Rotemberg, J.J. (1983). "Aggregate Consequences of Fixed Costs of Price Adjustment." American Economic Review, 73:June, pp. 433-6.

Rudd, J. and K. Whelan (2005). New Tests of the New-Keynesian Phillips Curve, Journal of Monetary Economics, vol. 52, pp. 1167-81.

Russell, B. (1998). A Rules Based Model of Disequilibrium Price Adjustment with Missing Information, Dundee Discussion Papers, Department of Economic Studies, University of Dundee, November, No. 91.

Russell, B. (2006). Non-stationary Inflation and the Markup: an Overview of the Research and some Implications for Policy, Dundee Working Papers, Department of Economic Studies, University of Dundee, no. 191.

Russell, B. (2007). Non-Stationary Inflation and Panel Estimates of United States Short and Long-run Phillips curves, Dundee Discussion Papers, Department of Economic Studies, University of Dundee, May, No. 200.

Russell, B., J. Evans, and B. Preston. (2002). "The Impact of Inflation and Uncertainty on the Optimum Markup set by Firms." European University Institute Working Paper, ECO No. 2002/2.

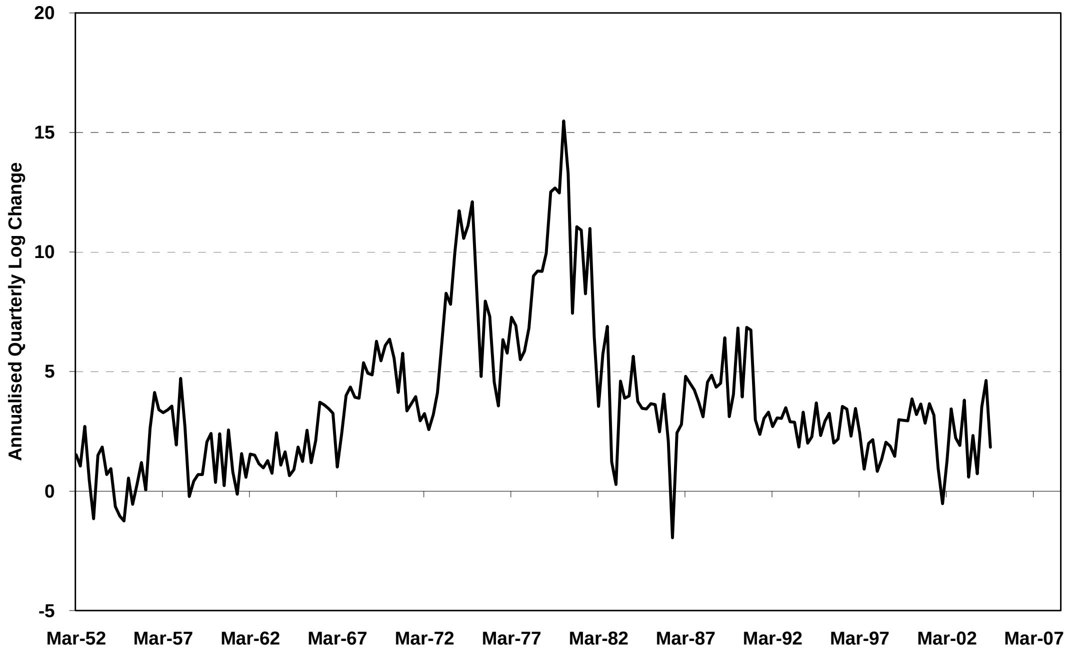
Staiger, D. and J.H. Stock (1997). Instrumental Variables Regression with Weak Instruments, Econometrica, vol. 65 (3), pp. 557-86.

Stock, J., and J. Wright (2000). GMM with Weak Identification, Econometrica, vol. 68, pp. 1097-1126.

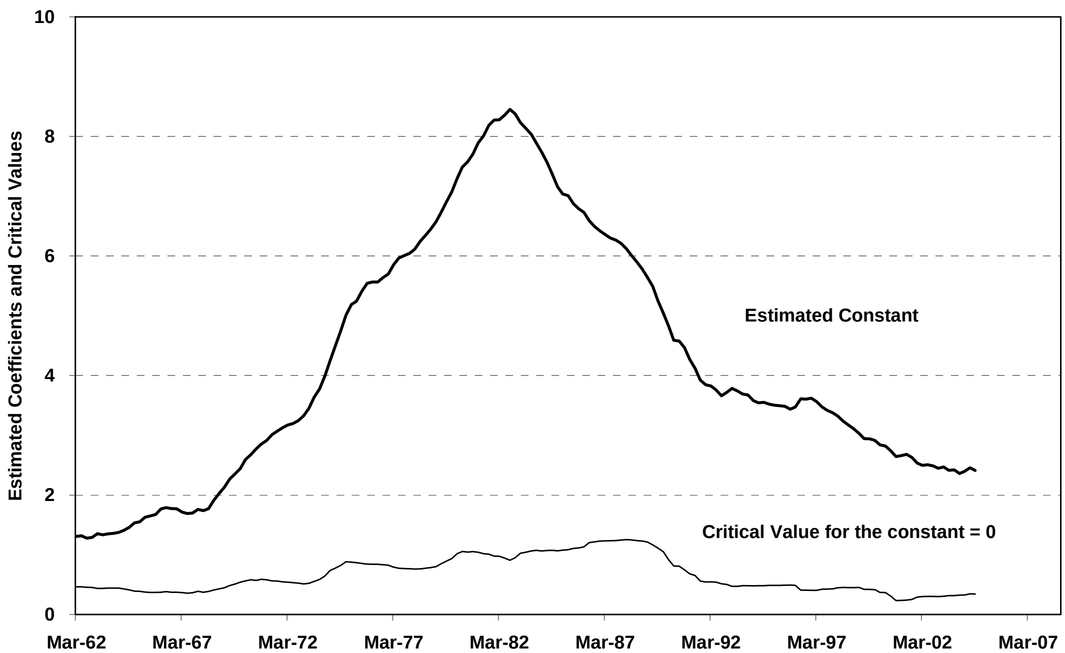
Stock, J., J. Wright, and M. Yogo (2002). A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments, Journal of Business and Economic Statistics, vol. 20 (4), pp. 518-30.

Svennson, L.E.O. (2000). Open Economy Inflation Targeting, Journal of International Economics, vol. 50, pp. 155-83.

Graph 1: United States CPI Inflation
March 1952 – September 2004

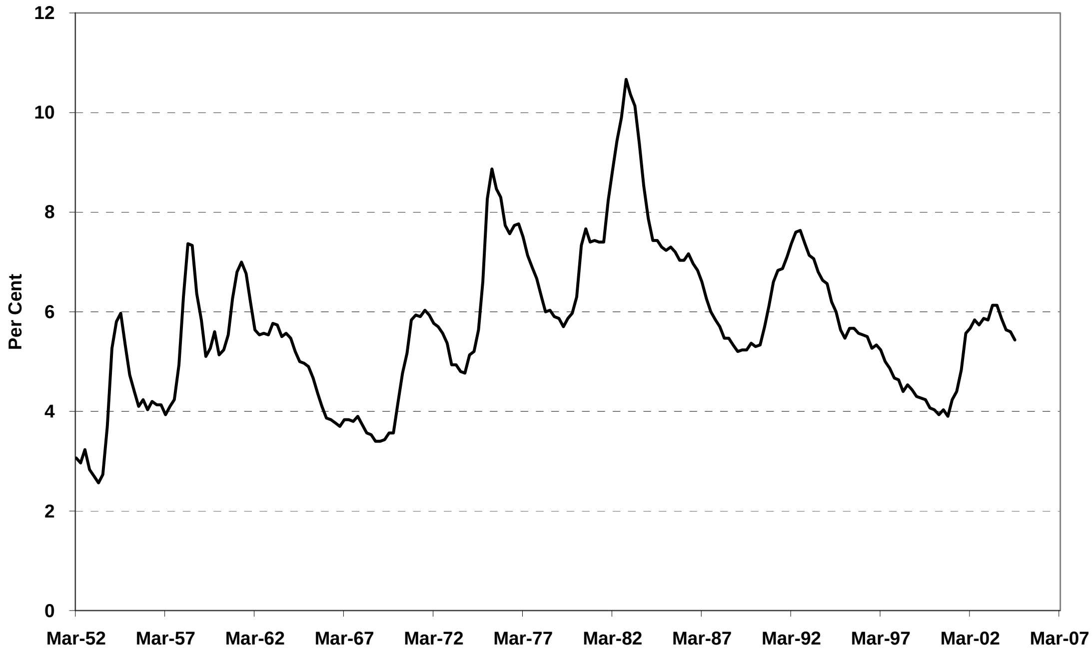


Graph 2: Estimated Constant from Rolling 10 Year CPI Inflation Regression

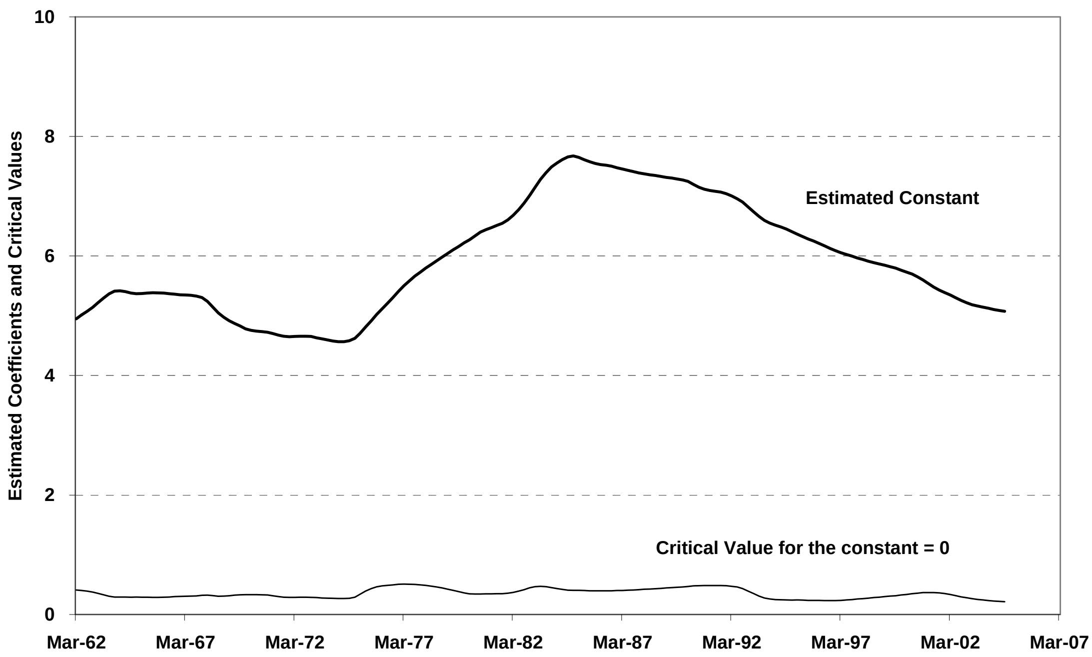


Note: The critical value shown in the graph is $1.96.\hat{\sigma}_{40}$ where $\hat{\sigma}_{40}$ is the standard error of the constant estimated from the rolling regressions.

Graph 3: United States Unemployment Rate
March 1952 – September 2004



Graph 4: Estimated Constant from Rolling 10 Year Unemployment Rate Regression

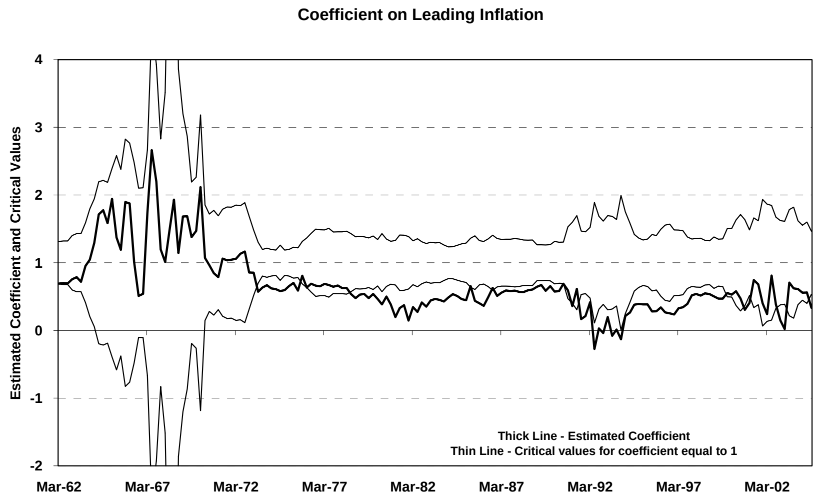


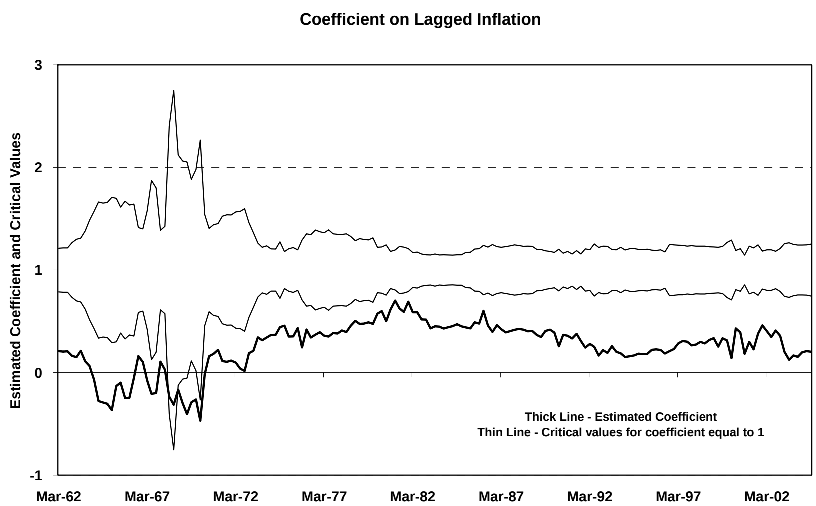
Note: See footnote to Graph 2.

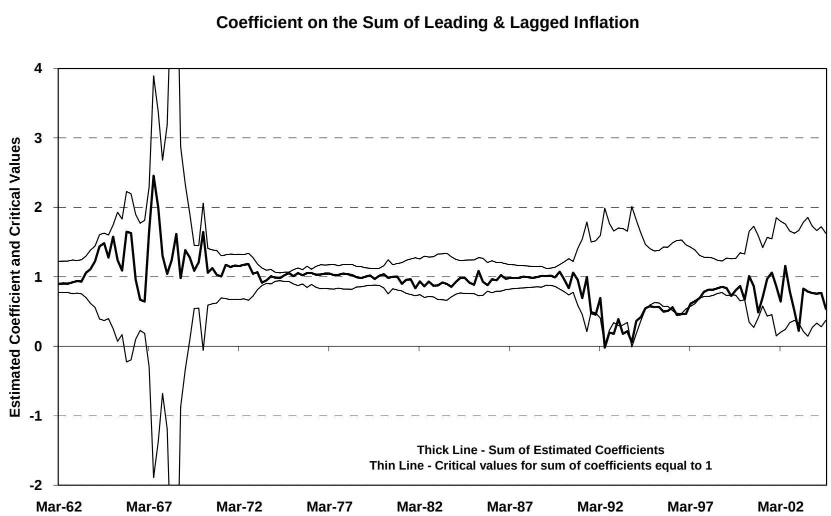
Graph 5: Mean Inflation and Unemployment Rates

Notes: The mean values of inflation and the unemployment rate are the estimated constants from the 10-year rolling regressions reported in Graphs 2 and 4.

Graph 6: GMM Rolling 10-Year Estimates of the Hybrid Phillips Curve Model



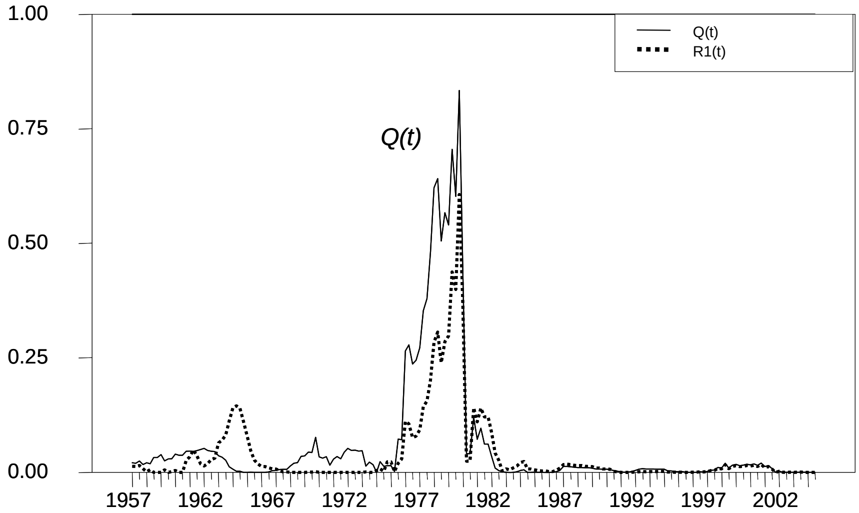




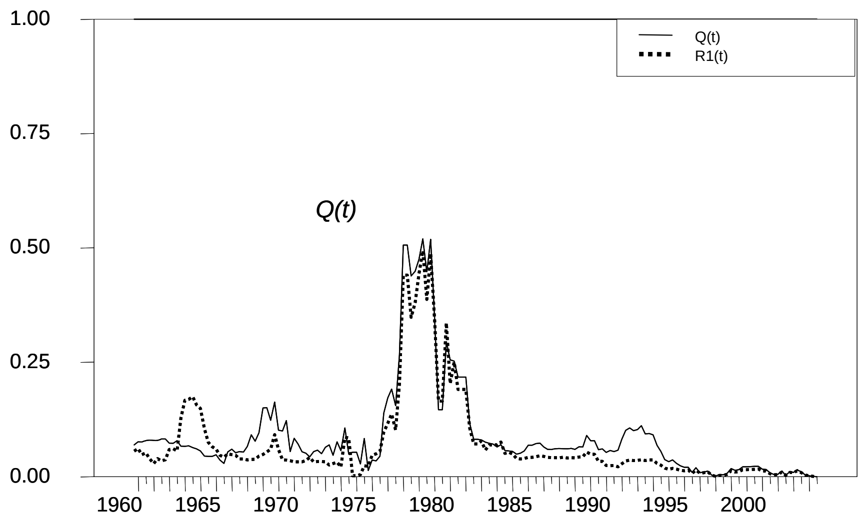
Note: The critical value shown in the graph is $1 \pm 1.96.\hat{\sigma}_{40}$ where $\hat{\sigma}_{40}$ is the standard error of $\hat{\delta}_f$ in the top panel, of $\hat{\delta}_b$ in the middle panel and of $\hat{\delta}_f + \hat{\delta}_b$ in the bottom panel.

Graph 7: Tests of the Stability of the Long-run Relationships

Two Variable I(1) System

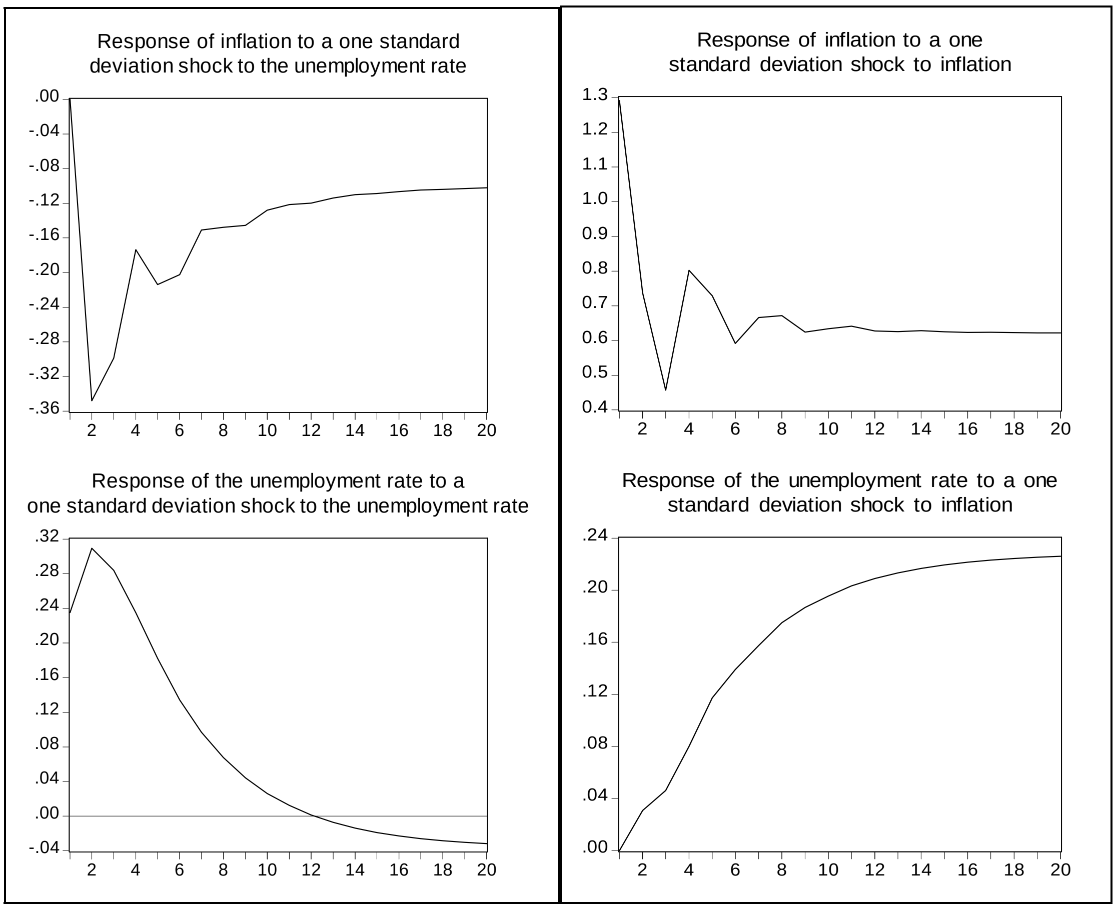


Three Variable I(1) System

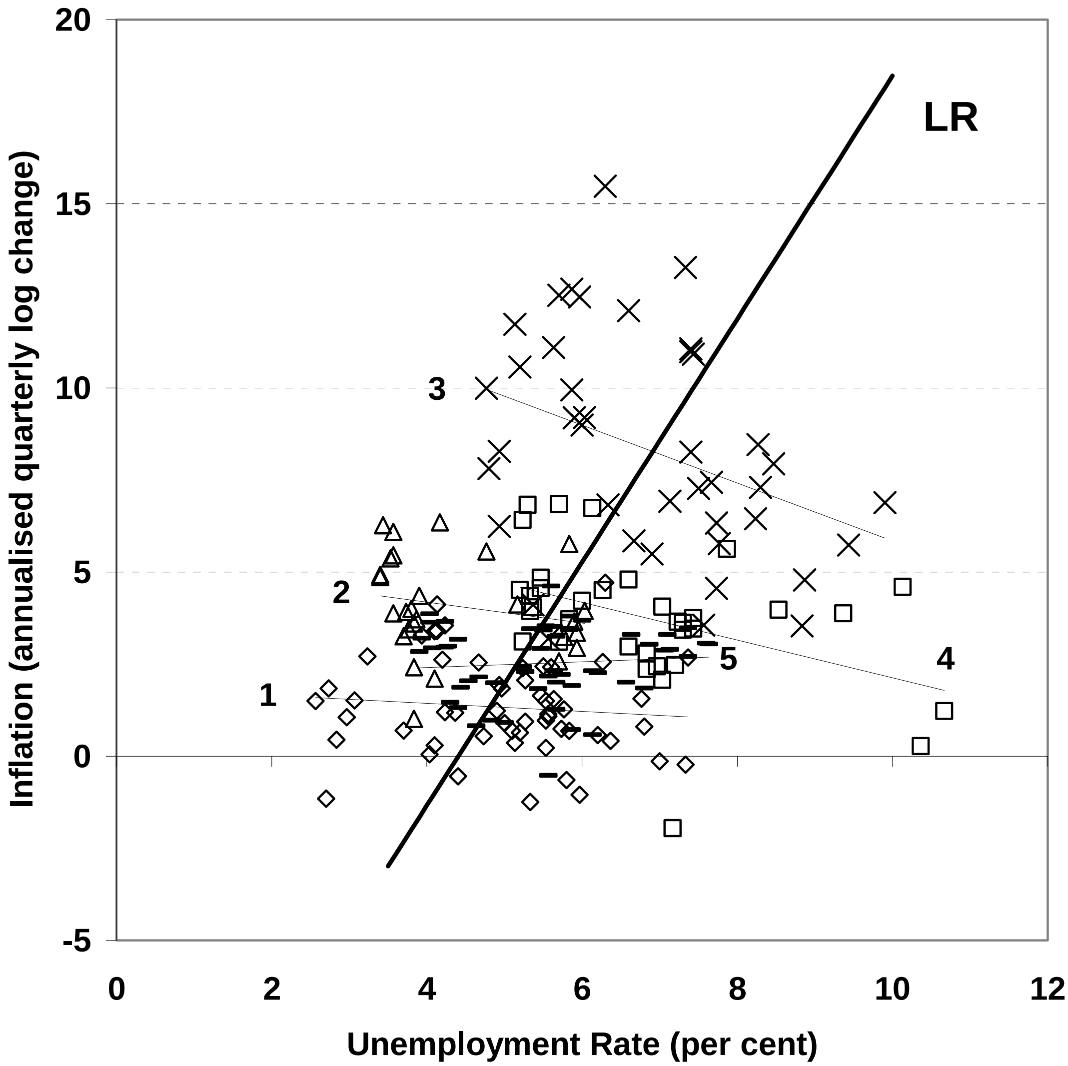


Notes: Recursive estimation is undertaken excluding the spike dummies. Q(t) is the statistic computed when all parameters of the model are re-estimated in every step of the recursive estimation. With R1(t) only the long-run parameters are re-estimated while the short-run parameters remain at their full sample estimates. The critical values are reported in Tables C10 to C13 of the CATS 2.0 manual and in the figures above a value of Q(t) in excess of 1 indicates a rejection of constancy of the cointegrating vector.

Graph 8: Impulse Response Functions of the Two Variable I(1) Model



Graph 9: United States Long-run Phillips Curve



	Inflation Episodes	Symbol	Mean Inflation
1	March 1952 to September 1965	Diamond	1.3
2	December 1965 to June 1972	Triangle	4.1
3	September 1972 to September 1982	Cross	8.3
4	December 1982 to June 1991	Square	3.8
5	September 1991 to September 2004	Dash	2.5

Table 1: Estimates of the United States Hybrid Phillips Curve
March 1952 to September 2004

Variable	Estimated Coefficient	Standard Error	*t* Statistic	$CV^{S}_{5\%}$	$CV^{RW}_{2.5\%}$	$CV^{RW}_{97.5\%}$	$CV^{0.9}_{2.5\%}$	$CV^{0.9}_{2.5\%}$
Δp_{t+1}	0.7055	0.0988	7.14	1.96	- 0.36	2.78	- 0.39	2.59
Δp_{t-1}	0.2946	0.0822	3.59	1.96	- 0.37	2.75	- 0.32	3.09
$u_t - u^*$	- 0.0238	0.0567	- 0.42	1.96				
δ	- 0.0305	0.1140	- 0.27	1.96				

Testing if coefficients are insignificantly different from 1

	Estimated Coefficient	Standard Error	*t* Statistic	$CV^{S}_{5\%}$	$CV^{RW}_{5\%}$	$CV^{0.9}_{5\%}$
$\delta_f - 1$	- 0.2945	0.0988	- 2.98	- 1.64	- 2.23	- 2.14
$\delta_b - 1$	- 0.7054	0.0822	- 8.58	- 1.64	- 2.37	- 2.66
$\delta_f + \delta_b - 1$	0.0001	0.0303	0.00	- 1.64	- 0.72	- 0.90

Diagnostic Tests

Lagrange multiplier test of serial correlation: $\chi_1^2 = 173.09$, p-value 0.0000; $\chi_4^2 = 202.85$, p-value 0.0000. ADF test of residuals: *t* statistic = - 8.17, $CV_{1\%}$ = - 2.58. J Test (Hansen 1982) of instrument validity: 12.43, p-value 0.0060, $CV_{1\%}$ $\chi_1^2 = 6.64$ and $CV_{5\%}$ $\chi_1^2 = 3.84$. $\bar{R}^2 = 0.79$. SEE = 1.3951.

Notes: The critical values shown in the table as CV^{RW} and $CV^{0.9}$ were calculated using Monte Carlo simulation techniques. That is, the same model with 209 observations is re-estimated 10,000 times and the histogram of the t-statistics used to compute the 95 per cent critical value.

Estimated model: $\Delta p_t = \delta_f \Delta p^e_{t+1} + \delta_b \Delta p_{t-1} + \delta_U (u_t - u^*) + \delta + \varepsilon_t$

Instruments: constant, three lags of Δp and three lags of $u_t - u^*$. Number of usable observations, 207. Data are quarterly and the unemployment rate is demeaned before estimation with a broken linear trend. Further details of the data are provided in the data appendix.

Table 2a: I(1) Inflation-Unemployment Rate Model

Normalised Cointegrating Vector	Δp	u
Long-run Relationship	1 (0.160)	- 2.714 (0.333)

Standard errors reported as ( ). Likelihood ratio tests on the long-run relationship (a) test of coefficient on inflation is zero is rejected, $\chi_1^2 = 31.41$, p-value = 0.00, (b) test of coefficient on the unemployment rate is zero is rejected, $\chi_1^2 = 50.65$, p-value = 0.00, and (c) test of the coefficient on the trend is zero is accepted $\chi_1^2 = 0.00$, p-value = 1.00.

Predetermined Variables: de-trended constant price GDP, spike dummies for periods where residuals are greater than 3 standard errors – March 1954 (recovery following the Korean War), December 1958 (world slowdown and historically high unemployment following the Second World War), March 1975 (OPEC first oil price rise and expansionary macro policies), September 1980 and December 1981 (OPEC second oil price rise and 'Volker deflation'), June 1986 (collapse of the OPEC oil cartel and the Plaza Accord).

Testing for the Number of Cointegrating Vectors

H_0: r =	**Eigenvalue**	**L-max**	**Trace**	**L-max_{90}**	**Trace_{90}**
0	0.2470	58.99	59.16	10.60	13.31
1	0.0008	0.17	0.17	2.71	2.71

The number of cointegration vectors is given by **r**. L-max_{90} and Trace_{90} are the relevant 90 per cent critical values for the L-max and Trace test statistics. Statistics computed with 3 lags of the core variables chosen by the significance of the last dynamic terms. Inference concerning the number of cointegrating vectors is not affected by the exclusion of the spike dummies. The sample is December 1952 to September 2004 and has 208 observations with 194 degrees of freedom.

System Diagnostics for the Model

(a) Tests for Serial Correlation

Ljung-Box (52) χ^2(198) = 211.744, p-value = 0.24

LM(1) χ^2 (4) = 9.143, p-value = 0.06

LM(4) χ^2(4) = 9.441, p-value = 0.05

(b) Test for Normality: Doornik-Hansen Test for normality: χ^2(4) = 5.941, p-value = 0.20.

Table 2b: Dynamics of the I(1) Inflation-Unemployment Rate Model

Equation ⇒ **Variable ⇓**	$\Delta^2 p$	Δu
Adjustment Coefficients Long-run Relationship	0.034 [1.3]	0.038 [8.2]
$\Delta^2 p_{t-1}$	- 0.463 [- 7.4]	- 1.389 [- 4.1]
$\Delta^2 p_{t-2}$	- 0.399 [- 6.7]	0.132 [0.4]
Δu_{t-1}	- 0.014 [- 1.2]	0.419 [6.8]
Δu_{t-2}	- 0.024 [- 2.2]	- 0.069 [- 1.2]
Business Cycle	8.484 [1.5]	- 5.982 [- 5.8]
Constant	0.494 [1.6]	0.448 [7.8]

Reported as [ ] are *t* statistics. The adjustment coefficients are the values with which the long-run relationship enters each equation of the system. The long-run relationship, or dynamic error correction term is therefore: $ECM_t \equiv \Delta p_t - 2.714 u_t$. The spike dummies referred to in Table 2a are not reported.

Table 3a: I(1) Inflation-Unemployment Rate-Markup Model

Normalised Cointegrating Vector	Δp	u	mu
Long-run Relationship 1	1 (0.158)	- 3.271 (0.376)	
Long-run Relationship 2	1 (0.176)		26.344 (5.088)

Standard errors reported as ( ). Likelihood ratio tests (a) Long-run Relationship 1 - test of coefficient on inflation is zero is rejected, $\chi_1^2 = 26.43$, p-value = 0.00, and the markup is zero is rejected, $\chi_1^2 = 19.28$, p-value = 0.00 (b) Long-run Relationship 2 - test of coefficient on the inflation is zero is rejected, $\chi_1^2 = 12.06$, p-value = 0.00, and (c) test of the coefficient on the trend is zero is accepted $\chi_1^2 = 1.58$, p-value = 0.45.

Predetermined Variables: de-trended constant price GDP, and spike dummies for periods where residuals are greater than 3 standard errors – March 1954 and March 1956 (recovery following the Korean War), March 1958 (world slowdown and historically high unemployment following the Second World War), March 1975 and June 1978 (OPEC first oil price rise and expansionary macro policies), September 1980, December 1981, March 1982 and December 1982 (OPEC second oil price rise and 'Volker deflation'), June 1986 (collapse of the OPEC oil cartel and the Plaza Accord), and March 2000 (end of the technology stock price 'bubble').

Testing for the Number of Cointegrating Vectors

H_0: r =	**Eigenvalue**	**L-max**	**Trace**	**L-max$_{90}$**	**Trace$_{90}$**
0	0.2744	66.41	102.26	13.39	26.70
1	0.1508	33.84	35.85	10.60	13.31
2	0.0097	2.01	2.01	2.71	2.71

The number of cointegration vectors is given by **r**. L-max$_{90}$ and Trace$_{90}$ are the relevant 90 per cent critical values for the L-max and Trace test statistics. Statistics computed with 4 lags of the core variables chosen by the significance of the last dynamic terms. Inference concerning the number of cointegrating vectors is not affected by the exclusion of the spike dummies. The sample is March 1953 to September 2004 and has 207 observations with 182 degrees of freedom.

System Diagnostics for the Model

(a) Tests for Serial Correlation

Ljung-Box (51) χ^2(426) = 426.949, p-value = 0.48

LM(1) χ^2 (9) = 12.295, p-value = 0.20

LM(4) χ^2(9) = 12.358, p-value = 0.19

(b) Test for Normality: Doornik-Hansen Test for normality: χ^2(6) = 7.126, p-value = 0.31.

Table 3b: Dynamics of the I(1) Inflation-Unemployment Rate - Markup Model

Equation ⇒ **Variable** ⇓	$\Delta^2 p$	Δu	Δmu
Adjustment Coefficients Long-run Relationship 1	- 0.005 [- 0.2]	0.030 [7.5]	- 0.000 (- 3.9)
Adjustment Coefficients Long-run Relationship 2	0.026 [0.7]	0.014 [2.2]	0.001 [5.7]
$\Delta^2 p_{t-1}$	- 0.379 [- 5.1]	- 1.336 [- 4.1]	- 35.700 [- 3.2]
$\Delta^2 p_{t-2}$	- 0.287 [- 4.1]	0.283 [0.8]	- 24.864 [- 1.8]
$\Delta^2 p_{t-3}$	0.064 [1.0]	0.070 [0.2]	- 12.877 [- 0.9]
Δu_{t-1}	- 0.018 [- 1.4]	0.401 [6.8]	- 0.324 [- 0.1]
Δu_{t-2}	- 0.037 [- 2.9]	0.008 [0.1]	- 5.101 [- 2.0]
Δu_{t-3}	- 0.014 [- 1.2]	- 0.179 [- 3.2]	- 0.552 [- 0.225]
Δmu_{t-1}	0.000 [1.5]	0.002 [1.4]	- 0.207 [- 3.328]
Δmu_{t-2}	- 0.000 [- 0.3]	0.002 [1.4]	0.103 [1.6]
Δmu_{t-3}	0.001 [1.9]	0.002 [1.5]	- 0.140 [- 2.2]
Business Cycle	9.662 [1.8]	- 4.790 [- 4.9]	0.045 [1.8]
Constant	- 3.781 [- 0.7]	- 1.711 [- 1.7]	- 0.150 [- 5.8]

Reported as [ ] are *t* statistics. The adjustment coefficients are the values with which the long-run relationship enters each equation of the system. The two long-run relationship, or dynamic error correction term are therefore: $ECM\,1_t \equiv \Delta p_t - 3.271 u_t$ and $ECM\,2_t \equiv \Delta p_t + 26.344\, mu_t$. The spike dummies referred to in Table 3a are not reported.