

**A MULTIPLE BREAK PANEL APPROACH TO ESTIMATING  
UNITED STATES PHILLIPS CURVES**

**DATA APPENDIXES, TABLES  
AND GRAPHS**

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## APPENDIX 1 DATA APPENDIX

The United States data are seasonally adjusted and quarterly for the period March 1960 to June 2007. The United States national accounts data are from the National Income and Product Account tables from the United States of America, Bureau of Economic Analysis. The data were downloaded via the internet on 9 October 2007. The United States and the Monte Carlo data are available at [www.BillRussell.info](http://www.BillRussell.info).

United States Data	
<i>Variable</i>	<i>Details</i>
GDP implicit price deflator at factor cost	Nominal GDP at factor cost is nominal GDP (Table 1.1.5, line 2) plus subsidies (NIPA Table 1.10, line 10) less taxes (NIPA Table 1.10, line 9). GDP implicit price deflator is nominal GDP at factor cost divided by constant price GDP at 2000 prices (NIPA Table 1.1.6, line 1). Inflation is the first difference of the natural logarithm of the GDP implicit price deflator at factor cost. Note that Graph 1 shows the estimated inflation regimes multiplied by 400 to be consistent with annualised inflation data.
The Markup	Calculated as the natural logarithm of nominal GDP at factor cost divided by wages, salaries and supplements (NIPA Table 1.10, line 2).

### The Data Generated for the Monte Carlo Analysis

The data are generated using WinRATS pro 6.2 and 7.2. The forcing variable,  $x_t$ , is generated as:  $x_t = 0.937967x_{t-1} + \omega_t$  where the first observation,  $x_0$ , is zero and  $\omega_t$  is a random draw from a normal distribution with mean zero and a standard error of 0.006388. The 'seed' value is: 250305.

The 'inflation' series,  $y_t$ , is generated as:  $y_t = -0.205406x_t + v_t$  where  $v_t$  is a random draw from a normal distribution with a mean of zero and a standard error of 0.004753. The 'seed' value is: 171193.

The mean-shift 'inflation' variable,  $y_t^{MS}$ , is:  $y_t^{MS} = y_t + \mu_t^i$  where  $\mu_t^i$  is the mean rate of inflation in regime  $i$  as reported in Table A2 of Appendix 2.

## APPENDIX 2 IDENTIFYING THE INFLATION REGIMES

The Bai and Perron (1998, 2003a, 2003b) approach minimises the sum of the squared residuals to identify the dates of  $k$  breaks in the inflation series and, thereby, identify  $k+1$  ‘inflation regimes’. The estimated model is:

$$\Delta p_t = \gamma_{k+1} + \tau_t \quad (\text{A2.1})$$

where  $\Delta p_t$  is inflation and  $\gamma_{k+1}$  is a series of  $k+1$  constants that estimate the mean rate of inflation in each of  $k+1$  inflation regimes and  $\tau_t$  is a random error. The model is corrected for serial correlation with a minimum regime size (or ‘trimming rate’) of 5 per cent of the total sample (nine quarters). The final model is chosen using the Bayesian Information Criterion. If the model is not corrected for serial correlation the break dates are identical. The model is estimated using quarterly data for the period March 1960 to June 2007 for the United States. The results of the estimated model are reported in the table below. Note that Graph 1 shows the estimated inflation regimes multiplied by 400 to be consistent with annualised inflation data. The Bai-Perron technique was estimated using Gauss 5.0 and the programme was kindly made available by Pierre Perron on his personal internet site.

<i>Regime</i>	<i>Dates of the ‘Inflation Regimes’</i>	<i>Mean Rate of Inflation</i>
1	March 1960 to September 1964	0.003133
2	December 1964 to June 1967	0.006844
3	September 1967 to December 1972	0.011385
4	March 1973 to March 1975	0.021266
5	June 1975 to June 1977	0.015419
6	September 1977 to September 1981	0.020361
7	December 1981 to December 1990	0.008863
8	March 1991 to September 2003	0.005005
9	December 2003 to June 2007	0.007613

**Table 1: Phillips Curve estimates from the generated data**

	Constant Mean Rate of Inflation dependent variable $y_t$					Shifting Mean Rates of Inflation dependent variable $y_t^{MS}$		
	F-P	NK	Hybrid	ND		F-P	NK	Hybrid
$y_{t+1}$		0.0191 (0.0)	0.0186 (0.0)		$y_{t+1}^{MS}$		1.0315 (14.1)	0.9785 (5.2)
$y_{t-1}$	-0.0104 (-0.2)		-0.0078 (-0.1)		$y_{t-1}^{MS}$	0.2984 (4.4)		0.0377 (0.6)
					$y_{t-2}^{MS}$	0.2201 (3.1)		
					$y_{t-3}^{MS}$	0.1923 (3.1)		
$x_t$	-0.2076 (-8.1)	-0.2017 (-2.5)	-0.2034 (-2.1)	-0.2052 (-10.1)	$x_t$	-0.0563 (-1.8)	-0.0146 (-0.6)	-0.0158 (-0.6)
C	-0.0000 (-0.0)	-0.0000 (-0.0)	-0.0000 (-0.0)	-0.0000 (-0.0)	C	0.0027 (4.0)	-0.0003 (-0.4)	-0.0002 (-0.1)
$R^2$	0.76	0.77	0.83	0.70	$\bar{R}^2$	0.75	0.74	0.80
J test	0.4920	0.5185	0.5268	0.4964	J test	0.2911	0.4890	0.4835
LM(4)	0.4357	0.0746	0.0196	0.4478	LM(4)	0.1261	0.0000	0.0000
DW	1.99	2.02	2.01	2.00	DW	2.03	2.90	2.94
ADF <sub>R</sub>	-6.15	-6.55	-6.50	-6.12	ADF <sub>R</sub>	-5.92	-8.35	-8.43
$\sum$	-0.0104 [0.0656]	0.0191 [0.4795]	0.0108 [0.6472]		$\sum$	0.7108 [0.0699]	1.0315 [0.0769]	1.0161 [0.0947]
$F$	0.4230	0.4091	0.4392	0.4524	$F$	0.0000	0.0000	0.0000

Reported as ( ) and [ ] are  $t$ -statistics and standard errors respectively. The ‘Constant Mean Rate of Inflation’ models are estimated with the constructed inflation series,  $y_t$ , and the constructed forcing variable,  $x_t$ . The ‘Shifting Mean Rate of Inflation’ models are estimated with the constructed mean-shift inflation series,  $y_t^{MS}$ , and the forcing variable  $x_t$ . See Sections 2.2 and 2.3 for details of how the data are generated. The models are estimated with 190 observations using GMM with three lags of the dependent variable and the forcing variable as instruments. Further lags of the dependent variable and the forcing variable in the Friedman-Phelps and hybrid models are excluded on a 5%  $t$ -criterion. Reported as  $R^2$  is the pseudo  $R^2$ . Reported as J Test is the significance of the Hansen test of instrument validity, LM(4) is the significance of the fourth order autocorrelation Lagrange multiplier test statistic, DW is the Durbin-Watson test statistic, and ADF<sub>R</sub> is the no intercept and no trend ADF test of the residuals where the 1%, 5% and 10% critical values are -2.576, -1.941 and -1.616 respectively.  $\sum$  is the sum of the generated ‘dynamic inflation terms’.  $F$  is the F-test probability value that the estimated parameters are equal to their ‘true’ values of  $\delta_f=0$ ,  $\delta_b=0$ , and  $\delta_x=-0.205406$  in the data generating process. 10,000 Monte Carlo models estimated with WinRATS pro 6.2.

**Table 2: Phillips Curve estimates from the differenced generated data**

	F-P	NK	Hybrid
$\Delta y_{t+1}^{MS}$		- 0.3772 (- 0.5)	- 0.4464 (- 0.7)
$\Delta y_{t-1}^{MS}$	- 0.6013 (- 6.6)		- 0.6006 (- 4.6)
$\Delta y_{t-2}^{MS}$	- 0.2869 (- 3.3)		- 0.3005 (- 2.3)
$\Delta x_t$	0.0129 (- 0.8)	- 0.2089 (- 0.3)	0.0375 (- 0.1)
Constant	0.0000 (0.1)	0.0000 (0.5)	0.0001 (0.1)
$\bar{R}^2$	0.71	0.56	0.84
J test	0.2397	0.2298	0.3955
LM(4)	0.0405	0.0000	0.0029
DW	2.05	2.44	2.07
ADF <sub>R</sub>	-7.13	- 7.54	- 7.23
$\sum$	- 0.8882 [0.1890]	- 0.3772 [1.1864]	- 1.3475 [5.3564]
$F$	0.0052	0.1725	0.2447

The  $\Delta$  symbol represents the lag difference such that  $\Delta y_t = y_t - y_{t-1}$ . The dependent variable is  $\Delta y_t^{MS}$ . Further lags of  $\Delta y^{MS}$  and  $\Delta x$  in the Friedman-Phelps and hybrid models are excluded on a 5%  $t$ -criterion. See the notes to Table 1 for further details concerning this table.

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**Table 3: Monte Carlo Bai-Perron Estimates of the Inflation Regimes**

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Estimated Number of Breaks $k$	Implied Number of Inflation Regimes	Frequency
1	2	3
2	3	365
3	4	1146
4	5	2286
5	6	2768
6	7	1893
7	8	1037
8	9	387
9	10	115

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Statistical analysis of the number of breaks  $k$ .

Mean: 4.99, Median: 5, Standard Deviation: 1.469, Skewness: 0.225, Kurtosis: - 0.194.

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The number of breaks  $k$  is estimated in the model:  $y_t^{MS} = \gamma_{k+1} + \tau_t$  using the Bai-Perron technique where  $y_t^{MS}$  is the generated mean shift inflation variable,  $\gamma_{k+1}$  is a series of  $k+1$  constants that estimate the mean rate of inflation in each of  $k+1$  inflation regimes and  $\tau_t$  is a random error. Frequency is the number of  $y_t^{MS}$  series that have the estimated number of breaks. The 'true' number of breaks in the series is 8 implying 9 inflation regimes. See Appendix 2 for further details concerning the Bai-Perron technique for estimating structural breaks. Bai-Perron technique estimated using Gauss 5.0 assuming a minimum regime size of 9 periods.

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**Table 4: Monte Carlo Panel Estimates of the Phillips Curve using the Generated Mean Shift Variable  $y_t^{MS}$  and the Forcing Variable  $x_t$**

	Restricted Constant			Fixed Effects			
	F-P	NK	Hybrid	F-P	NK	Hybrid	ND
$y_{t+1}^{MS}$		0.9683 (8.0)	0.9400 (4.4)		0.1567 (0.3)	0.1408 (0.3)	
$y_{t-1}^{MS}$	0.5198 (8.0)		0.0260 (0.4)	0.0211 (0.3)		0.0188 (0.2)	
$x_t$	-0.0920 (-2.5)	-0.0237 (-0.5)	-0.0230 (-0.5)	-0.1095 (-2.5)	-0.1060 (-1.6)	-0.1100 (-1.4)	-0.1113 (-2.7)
Constant	0.0044 (5.6)	0.0003 (0.2)	0.0003 (0.3)	0.0090 (24.1)	0.0077 (18.9)	0.0077 (18.4)	0.0092 (24.6)
$\bar{R}^2$	0.415	0.231	0.216	0.599	0.477	0.433	0.598
LM(1)	[0.007]	[0.000]	[0.000]	[0.588]	0.061	[0.025]	[0.355]
LM(2)	[0.011]	[0.000]	[0.000]	[0.540]	[0.076]	[0.035]	[0.394]
LM(3)	[0.009]	[0.000]	[0.000]	[0.529]	[0.084]	[0.038]	[0.412]
LM(4)	[0.006]	[0.000]	[0.000]	[0.521]	[0.085]	[0.039]	[0.420]
DW	2.339	2.928	2.954	2.004	2.169	2.171	1.960
<i>Wald Tests – probability values</i>							
$\phi_f + \phi_b = 0$	[0.000]	[0.000]	[0.000]	[0.662]	[0.724]	[0.845]	
$\phi_f + \phi_b = 1$	[0.000]	[0.880]	[0.934]	[0.000]	[0.343]	[0.528]	
$W$	[0.000]	[0.000]	[0.000]	[0.164]	[0.218]	[0.255]	[0.141]
<i>F Tests – probability values</i>							
Significant Variables	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Fixed Effects				[0.000]	[0.001]	[0.000]	[0.000]

Reported as ( ) and [ ] are  $t$ -statistics and probability values respectively. The forcing variable is,  $x_t$ , and the dependent variable is,  $y_t^{MS}$ . The data are partitioned into inflation regimes as estimated by the Bai-Perron technique. Details of the number of breaks found in the inflation series are reported in Table 4. The cross section models are then estimated using the 2SLS estimator with three lags of  $x$ , and,  $y^{MS}$  as instruments. The process is repeated 10,000 times using Monte Carlo techniques with GAUSS 5.0. The data used in the analysis is identical to that used in the estimation of the models reported in Tables 1 and 2. In the first three columns the constant (or fixed effect) in each panel is restricted to be the same such that  $\phi^1 = \phi^2 = \dots = \phi^n$ . The fixed effects models in columns 4 to 6 the reported constant is the weighted average of the fixed effects. LM(1) to LM(4) are the Breusch-Pagan Lagrange multiplier tests of first to fourth order serial correlation in the residuals.  $W$  tests the estimates parameters are equal to their ‘true’ values of  $\delta_f = 0$ ,  $\delta_b = 0$ , and  $\delta_x = -0.205406$  in the data generating process. ‘Significant Variables’ tests  $\phi_f = \phi_b = \phi_z = \phi^n = 0$ . ‘Fixed Effects’ tests that the fixed effects are zero such that  $\phi^n = 0$ .

**Table 5: Panel Estimates of United States Phillips Curve**

	All Inflation Regimes								Stationary Inflation Regimes			
	Restricted Constant				Fixed Effects				Fixed Effects			
	F-P	NK	Hybrid	Markup Only	F-P	NK	Hybrid	Markup Only	F-P	NK	Hybrid	Markup Only
	1	2	3	4	5	6	7	8	9	10	11	12
$\Delta p_{t+1}^n$		0.9835 (14.4)	0.6888 (5.6)			0.0636 (0.2)	0.3819 (1.0)			0.2392 (0.5)	0.4186 (0.9)	
$\Delta p_{t-1}^n$	0.4642 (6.1)		0.2754 (2.7)		0.1263 (1.6)		0.1748 (1.8)		0.0573 (0.7)		0.0845 (0.8)	
$\Delta p_{t-2}^n$	0.1477 (1.8)											
$\Delta p_{t-3}^n$	0.2805 (3.6)											
$mu_t$	- 0.0409 (- 2.6)	- 0.0064 (- 0.3)	- 0.0153 (- 0.9)	- 0.2106 (- 9.7)	- 0.0527 (- 2.5)	- 0.0571 (- 2.2)	- 0.0411 (- 1.5)	- 0.0581 (- 2.7)	- 0.0441 (- 2.2)	- 0.0438 (- 1.8)	- 0.0365 (- 1.4)	- 0.0469 (- 2.3)
Constant	0.0205 (2.6)	0.0032 (0.3)	0.0076 (0.9)	0.1094 (10.5)	0.0330 (3.3)	0.0356 (2.6)	0.0236 (1.5)	0.0367 (3.5)	0.0288 (2.9)	0.0272 (2.1)	0.0215 (1.4)	0.0306 (3.2)
$\bar{R}^2$	0.786	0.711	0.785	0.340	0.838	0.827	0.816	0.835	0.810	0.795	0.774	0.810
AR(1)	[0.031]	[0.000]	[0.000]	[0.000]	[0.844]	[0.575]	[0.000]	[0.195]	[0.429]	[0.012]	[0.000]	[0.708]
AR(2)	[0.144]	[0.020]	[0.455]	[0.000]	[0.020]	[0.033]	[0.119]	[0.024]	[0.065]	[0.090]	[0.152]	[0.068]
AR(3)	[0.088]	[0.668]	[0.668]	[0.000]	[0.760]	[0.821]	[0.728]	[0.626]	[0.546]	[0.227]	[0.245]	[0.555]
AR(4)	[0.068]	[0.197]	[0.151]	[0.000]	[0.551]	[0.542]	[0.285]	[0.729]	[0.399]	[0.305]	[0.292]	[0.403]
DW	2.121	2.769	3.027	0.485	2.048	1.886	2.665	1.82	2.051	2.378	2.747	1.94
<i>Wald Tests – probability values</i>												
Parameter Constancy	[0.000]	[0.209]	[0.383]	[0.000]	[0.134]	[0.336]	[0.128]	[0.413]	[0.253]	[0.669]	[0.393]	[0.261]
$\phi_f + \phi_b = 0$	[0.000]	[0.000]	[0.000]		[0.101]	[0.8426]	[0.197]		[0.481]	[0.527]	[0.326]	
$\phi_f + \phi_b = 1$	[0.044]	[0.809]	[0.545]		[0.000]	[0.004]	[0.303]		[0.000]	[0.046]	[0.332]	
<i>F Tests – probability values</i>												
Significant Variables	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Fixed Effects					[0.000]	[0.376]	[0.977]	[0.000]	[0.000]	[0.600]	[0.946]	[0.000]

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**Notes to Table 5**

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Reported as ( ) and [ ] are  $t$ -statistics and probability values respectively. The dependent variable is,  $\Delta p_t^n$  and the forcing variable is the markup,  $mu_t$ . The panels for ‘all the inflation regimes’ consist of 9 cross-sections with 190 observations in total and 160, 150 and 150 usable observations in the F-P, NK and hybrid models respectively. The ‘stationary inflation regimes’ include regimes 1, 2, 3, 6, 7, 8 and 9 with 151 observations. See appendices 1 and 2 for details concerning the data and the estimation of the inflation regimes. Lag length chosen by lag exclusion F-tests in all models except the restricted constant markup only model in column 4 where further dynamics do not improve the system diagnostics. Instruments are three lags of the independent variables in all models. Inference is not affected by the inclusion of fewer or more lags of the instruments. In columns 1 to 4 the constant (or fixed effect) in each panel is restricted to be the same such that  $\phi^1 = \phi^2 = \dots = \phi^9$ . In the fixed effects models in columns 5 to 12 the reported constant is the weighted average of the fixed effects. AR(1) to AR(4) are the Arellano-Bond tests of first to fourth order serial correlation in the residuals. ‘Parameter Constancy’ tests the estimated parameters for  $\Delta p_t^n$  and  $mu_t$  are the same across inflation regimes. ‘Significant Variables’ tests  $\phi_f = \phi_b = \phi_z = \phi^n = 0$ . ‘Fixed Effects’ tests the fixed effects are zero such that  $\phi^n = 0$ . Models estimated with 2SLS using Stata/SE 8.2 and Eviews 5.1.

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**Table 6: Estimates of the Long-run Phillips Curve**

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*Linear:* 
$$\overline{\Delta p} = 0.1109 - 0.2120 \bar{z}, \bar{R}^2 = 0.32$$
  
(14.3)      (-13.4)

The estimated coefficient on  $\bar{z}$  is zero is rejected,  $\chi_1^2 = 179.1191$ , prob-value = 0.0000. Standard error of the regression: 0.0049.

*Non-linear Exponential Model* 
$$\text{Ln}(\overline{\Delta p}) = 5.8436 - 22.2860 \bar{z}, \bar{R}^2 = 0.34$$
  
(2.8)      (-5.1)

The estimated coefficient on  $\bar{z}$  is zero is rejected,  $\chi_1^2 = 26.1511$ , prob-value = 0.0000. Standard error of the regression: 0.4920.

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Notes: Numbers in ( ) are  $t$  statistics . The models are estimated using ordinary least squares in Eviews 7.1 with Newey-West HAC standard errors on 7 combinations of the long-run rate of inflation and long-run markup calculated from column 12 of Table 5.

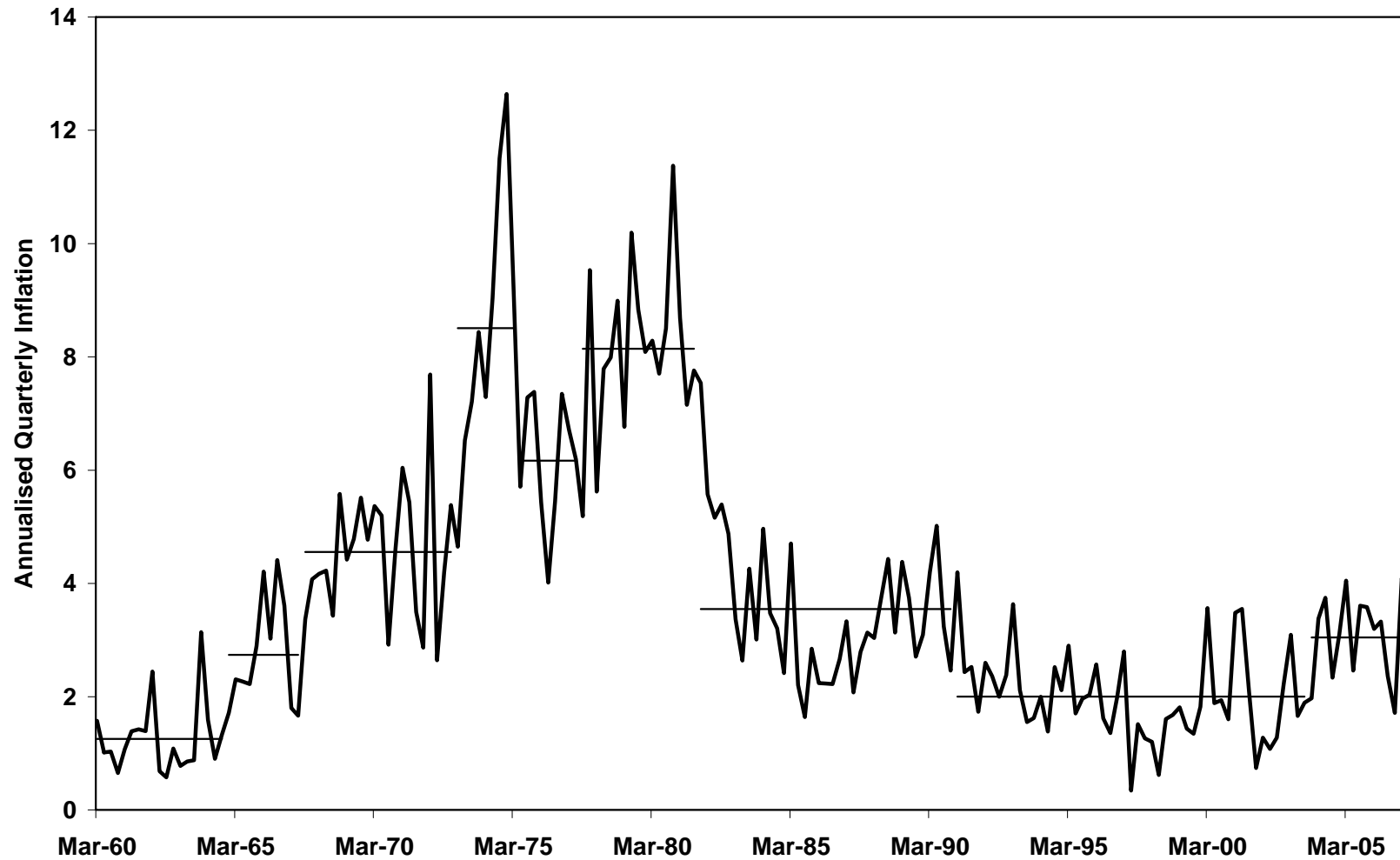
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**Table 7: Estimates of the Hybrid United States Phillips Curves  
Data from Cogley and Sbordone 2008**

Time Series	Panel Estimation			
	Restricted Constant		Fixed Effects	
$\Delta p gap_{t+1}$	0.9327 (4.0)	$\Delta p gap_{t+1}^n$	1.0839 (4.6)	0.0696 (0.1)
$\Delta p gap_{t-1}$	0.0224 (0.1)	$\Delta p gap_{t-1}^n$	0.0207 (0.1)	- 0.0073 (- 0.1)
$mu gap_t$	-0.0391 (- 0.6)	$mu gap_t^n$	0.0021 (0.1)	- 0.0347 (0.6)
Constant	- 0.0001 (- 0.1)	Constant	0.0000 (0.0)	0.0040 (1.7)
$\bar{R}^2$	0.503		0.4980	0.626
DW	2.895		3.051	2.124
<i>Wald Tests – probability values</i>				
$\phi_f + \phi_b = 0$	[0.000]*		[0.000]	[0.898]
$\phi_f + \phi_b = 1$	[0.537]*		[0.325]	[0.109]

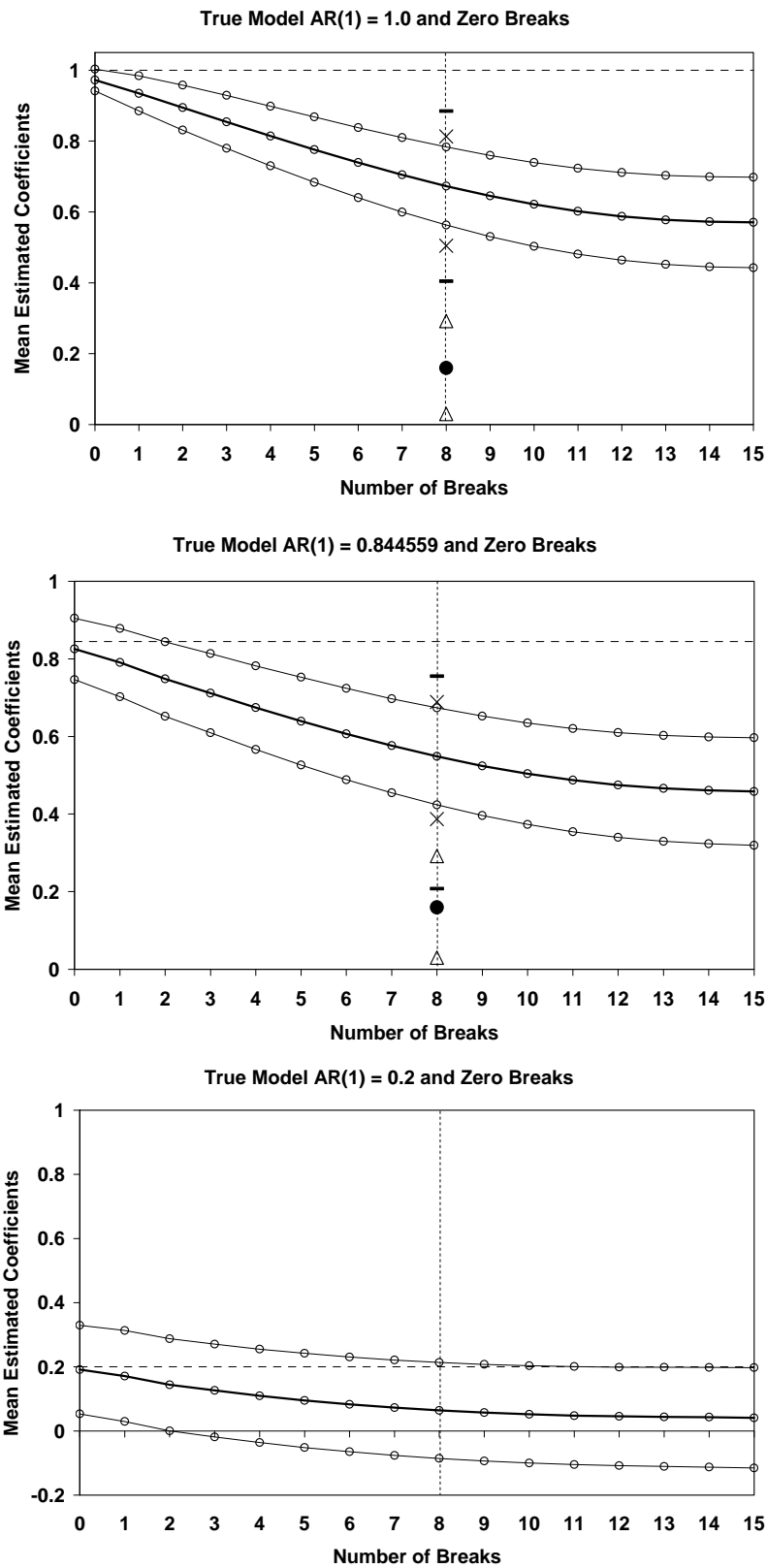
Reported as ( ) and [ ] are *t*-statistics and probability values respectively. Column 1 report time series estimates. Columns 2 and 3 report panel estimates where the data are partitioned in line with breaks identified with the Bai-Perron technique. The inflation, trend inflation and labour's income share data are from Cogley and Sbordone (2008) for the period March 1960 to June 2003. The inflation gap,  $\Delta p gap$ , is inflation less trend inflation. The markup gap,  $mu gap$ , is the negative of labour's income share less the BVAR estimate of labour's income share. The dependent variable is the inflation gap and the forcing variable is the markup gap. The time series contain 170 observations. The panels consist of 9 cross-sections with 138 usable observations after allowing for instruments. Instruments are three lags of the independent variables for all models. Inference is not affected by the inclusion of fewer or more lags of the instruments. Models estimated with 2SLS using Eviews 5.1. \* indicates F-test instead of Wald test. See also notes to Graph 3.

**Graph 1: United States Quarterly Inflation, Seasonally Adjusted, March 1960 – June 2007**



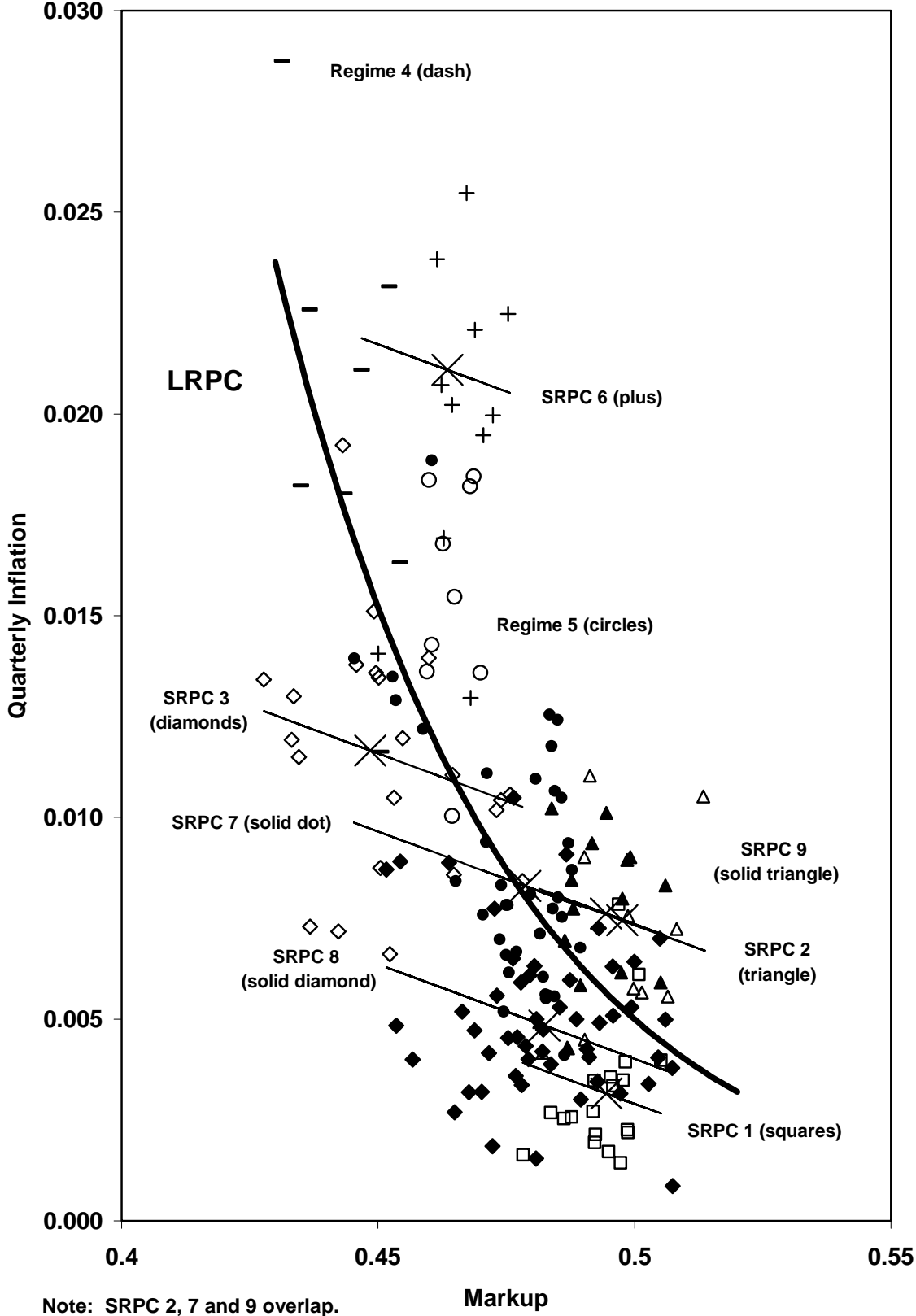
Notes: Horizontal dashed lines indicate the nine inflation regimes identified by the Bai-Perron technique (see Appendix 2 for details). Annualised quarterly inflation is measured as the change in the natural logarithm of the price index multiplying by 400.

**Graph 2: The Impact of Over-breaking on Estimates of the AR(1) Coefficients**

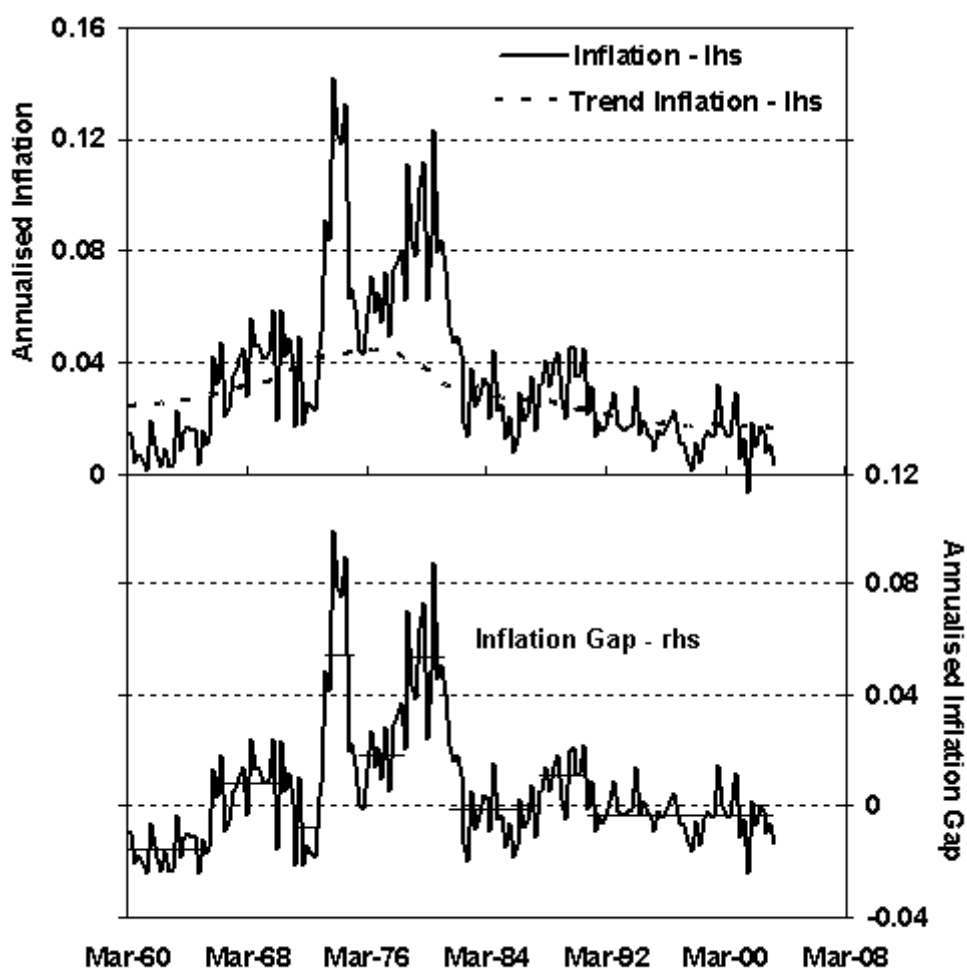


Notes: The dashes and crosses represent the maximum and minimum values and the first and 99<sup>th</sup> percentile estimates from the estimated AR(1) coefficients assuming eight breaks. The solid dot is the estimated United States inflation persistence of 0.1596 and the triangles delineate the 95 per cent confidence interval of the estimated persistence.

Graph 3: United States Inflation and the Markup



Graph 4: Cogley and Sbordone AER 2008 Inflation Data



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Notes: In the top panel the data are the same as that reported in Figure 1 of Cogley and Sbordone (2008). Inflation is measured as the quarterly change in the natural logarithm multiplied by four (to give the annualised rate) of the implicit gross domestic product deflator at market prices reported in Table 1.3.4 in the National Income and Product Account published by the United States Bureau of Economic Analysis. This is in contrast with our measure which is the same deflator but at factor cost to remove the direct effects on inflation and the markup of changes in indirect taxes and subsidies. While the later is theoretically appealing, in practice the factor cost adjustment has little impact on the estimates. Trend inflation is the first stage BVAR estimate of inflation. See also the notes to Table 7 and Cogley and Sbordone (2008) for more detailed information concerning the data. In the bottom panel the inflation gap is calculated as inflation less trend inflation from the top panel and is the same as that used in the second stage of the Cogley and Sbordone (2008). The horizontal thin lines in the bottom panel represent the mean inflation gap for each episode as identified using the Bai-Perron technique.

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